Stat/Biostat 578 flyer
MEASURE THEORY, for STATISTICIANS, Spring 2013

by Galen R. Shorack
Stat/Biostat 578:
MEASURE THEORY for STATISTICIANS

Instructor...Galen R. Shorack
Time...Spring Quarter 2013: 11:30–12:20 MWF
Text... “Measure Theory for Statisticians” by G. Shorack. This text is available only at the University Book Store, at the start of Spring Quarter. It is essentially a minor rewrite of Chapters 1-5, and bits of Chapters 7-8 of “Probability for Statisticians” by Galen R. Shorack (2000) Springer.
Prerequisites...The most important prerequisite is mathematical maturity, and this is typically enhanced by having some background in a “serious” math class. Math 574 at the UW should be barely sufficient; it is often taught from “Mathematical Analysis” by T. Apostol (1974) Addison–Wesley. (Chapters 1-4, and maybe a bit of Chapter 8 will be the content of Math 574 this year. Some of the material in Chapters 5 and 9 of Apostol will certainly be used in Stat 578.) An upper-division undergraduate math class in real analysis at another university could also be barely sufficient or sufficient.
Specific background needed... This includes open sets, closed sets, completeness, compactness, separability, equivalent metrics, continuity, convergence, and uniform convergence on a general metric space, as well as some of the major theorems that come with these concepts. Concepts such as the liminf and limsup of a sequence of real numbers $a_n$ are also used.

The course will focus on developing the basic results for the Lebesgue integral.
Chapter 1: Measures (constructing them, and determining which sets can be measured)
Chapter 2: Measurable functions (which functions can be measured (and later, integrated), and then study the convergence of such functions)
Chapter 3: Integration (define the integral of a function, and then determine the properties of integrals—such as the convergence of integrals when the functions being integrated also converge)
Chapter 4: Derivatives, via signed measures (Radon-Nikodym derivatives, or densities)
Chapter 5: Measures and processes on product spaces (or, joint distributions)
If time is available, there might be some discussion of a bit of:
Chapter 6: Distribution and quantile functions
Chapter 7: Independence and conditional expectation

The chapters 1-5 have a slight slant toward probability theory in their presentation. (The basic notation certainly reflects this.) But then, a probability space is just a measure space for which the measure of the total space is equal to 1. A few additional pages will be included, or handed out, that relate the measure theory results to results in probability theory and statistics. However, this is primarily a course that emphasizes the mathematical aspects.
My hope is that every student who enrolls will want to learn this material because they enjoy mathematics. My feeling is that those with other motives will find this course rather long.
Homework: Expect 5-8 problems per week. Typically, these consist of proving a result related to one presented in class, or considering an application of such a result.
Exams: One Midterm and the Final Exam.