Problem 1 – Campus conversations

Andrea is a genetics student who is doing her probability homework in the Atrium café. The conversation at a nearby table is distracting her. She cannot hear the words, but she would like to know what language the conversation is in.

What she knows is: on the UW campus there are English and Spanish speaking students (these are Andrea’s estimates of course). There are 3 times more English speaking students than Spanish speaking ones. When two students get together, if they have the same native language, they will talk in that language. Otherwise, they will talk in English with probability 4/5. Andrea also assumes that students pair in the Atrium independently of their native languages.

Denote by $E$, $\bar{E}$, $cE$, $\bar{c}E$ respectively the events “a student is a native English speaker”, “a student is a native Spanish speaker”, “the conversation is in English”, “the conversation is in Spanish”.

a. Help Andrea make an intelligent guess of the language. Compute for her the probability that the conversation is in English $P(cE)$. [Hint: Use the law of total probability.]

b. Andrea overhears the word “vista”. The probability of this word in Spanish is $P(\text{vista} | cE) = 3 \times 10^{-4}$; the probability of the same word in English is $P(\text{vista} | \overline{cE}) = 10^{-4}$. Compute the probability that the conversation is in English, given the overheared word $P(cE | \text{vista})$. Assume that the occurrence of “vista” is independent of anything else given the language.

c. Suppose that instead of “vista” Andrea overhears the word “cerveza”. The probability of this word in Spanish is $P(\text{cerveza} | cE) = 3 \times 10^{-4}$; the probability of the same word in English is $P(\text{cerveza} | \overline{cE}) = 10^{-6}$. Compute the probability that the conversation is in English, given the overheared word $P(cE | \text{cerveza})$. Compare with the result in b. and explain the difference. Assume that the occurrence of “cerveza” is independent of anything else given the language.

d. Andrea now recognizes one of the two people. She knows that he is an English speaker. What is the probability that the other person is a Spanish speaker, given that Andrea has overheard the word “cerveza”? [Hint: Start with Bayes’ rule. Note that “one of the two speakers is English” is not the same as “the first speaker is English.”]
Problem 2 (after Al. Drake)

Random variables $X, Y$ are described by the joint density

$$f_{XY}(x, y) = \begin{cases} K, & \text{if } x + y \leq 1, \ x, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

a. Make a drawing of this continuous outcome space.

b. Determine the constant $K$.

c. Are $X, Y$ independent random variables?

d. Determine $f_{X|Y}$

e. Determine $f_Y$.

f. Are $X, Y$ conditionally independent given event $C = \{\max(X, Y) \leq 0.5\}$?

g. Let $A = \{2(X - Y) \geq Y + X\}$, $B = \{Y > \frac{3}{4}\}$. Obtain the numerical values of $P(A)$, $P(B)$.

h. Determine and make a neatly labeled plot (by hand OK) of the density $f_{X|AB}$.

**Hint:** Represent probabilities as areas.