Exercise 3 of Assignment 1 (due 1/14/08)

Here we consider some basic properties of covariances. In what follows, assume for generality that all RVs and ordinary variables are complex-valued (the definition of covariance for complex-valued RVs is given in Section 2.5). All RVs are denoted by Z (or subscripted versions thereof), while c with or without a subscript denotes an ordinary variable. Note that, as usual, Z^* denotes the complex conjugate of Z and that $E\{Z^*\} = (E\{Z\})^*$.

(a) Show that $\operatorname{cov}\{Z, c\} = 0$.

(b) Show that $\operatorname{cov}\{Z_1, Z_2\} = (\operatorname{cov}\{Z_2, Z_1\})^*$.

(c) Show that $\operatorname{cov}\{Z_1 + c_1, Z_2 + c_2\} = \operatorname{cov}\{Z_1, Z_2\}$.

(d) Suppose that at least one of the RVs Z_1 and Z_2 has a zero mean. Show that $\operatorname{cov}\{Z_1, Z_2\} = E\{Z_1^* Z_2\}$.

(e) Show that

$$\operatorname{cov}\Big\{\sum_j c_{1,j} Z_{1,j}, \sum_k c_{2,k} Z_{2,k}\Big\} = \sum_j \sum_k c_{1,j}^* c_{2,k} \operatorname{cov}\{Z_{1,j}, Z_{2,k}\},$$

where the summations are over finite sets of integers.

Since real-valued variables can be regarded as a special case of complex-valued variables, the results above continue to hold when some or all of the variables in question are real-valued. In particular, when Z_1 and Z_2 are both real-valued, part (b) simplifies to $\operatorname{cov}\{Z_1, Z_2\} = \operatorname{cov}\{Z_2, Z_1\}$. Also, when Z_1 and $c_{1,j}$ in parts (d) and (e) are real-valued, we can simplify Z_1^* to Z_1, and $c_{1,j}^*$ to $c_{1,j}$.