Exercise 5 of Assignment 4 (due 2/4/08)

Let $\{\epsilon_t\}$ be a white noise process with mean zero and variance σ_ϵ^2. Define the stationary processes $\{X_t\}$ and $\{Y_t\}$ by

$$X_t = \frac{2}{9}\epsilon_t + \frac{5}{9}\epsilon_{t-1} + \frac{2}{9}\epsilon_{t-2}$$

and

$$Y_t = \frac{4}{9}\epsilon_t + \frac{4}{9}\epsilon_{t-1} + \frac{1}{9}\epsilon_{t-2}.$$

a) Let $L_X\{\cdot\}$ and $L_Y\{\cdot\}$ be LTI digital filters such that $L_X\{\epsilon_t\} = X_t$ and $L_Y\{\epsilon_t\} = Y_t$. Show that the gain function $|G_X(\cdot)|$ corresponding to $L_X\{\cdot\}$ is given by

$$|G_X(f)| = \frac{5}{9} + \frac{4}{9}\cos(2\pi f),$$

and find the gain function $|G_Y(\cdot)|$ corresponding to $L_Y\{\cdot\}$. In what respects do the associated transfer functions $G_X(\cdot)$ and $G_X(\cdot)$ differ?

b) Compare the sdf of $\{X_t\}$ with the sdf of $\{Y_t\}$.

c) How do $\{X_t\}$ and $\{Y_t\}$ differ from a moving average process as defined by Equation (43a)? Are there moving average processes that are equivalent to $\{X_t\}$ and $\{Y_t\}$ (i.e., have the same sdfs)?