Exercise 4 of Assignment 5 (due 2/11/08)

Suppose that $x_0, x_1, \ldots, x_{N-1}$ represents a time series and that we form its periodogram using

$$\hat{S}_x^{(p)}(f) = \frac{1}{N}\left|\sum_{t=0}^{N-1} x_t e^{-i2\pi ft}\right|^2$$

(see item [4] on page 205 for a comment on using the indexing $x_0, x_1, \ldots, x_{N-1}$ rather than our usual $x_1, x_2, \ldots, x_N$). For any integer $0 < m \leq N-1$, define $y_t = x_{t+m \bmod N}$; i.e., $y_0, y_1, \ldots, y_{N-1}$ is a circularly shifted version of $x_0, x_1, \ldots, x_{N-1}$ (see Exercise 4 of Assignment 1 for a discussion of the 'mod' operator). Let

$$\hat{S}_y^{(p)}(f) = \frac{1}{N}\left|\sum_{t=0}^{N-1} y_t e^{-i2\pi ft}\right|^2$$

be the periodogram for y_t.

a) Show that $\hat{S}_y^{(p)}(f_k) = \hat{S}_x^{(p)}(f_k)$, where $f_k = k/N$ is any one of the Fourier frequencies.

b) Prove or disprove the claim that, in general, $\hat{S}_y^{(p)}(f) = \hat{S}_x^{(p)}(f)$ at all frequencies (i.e., not necessarily just the Fourier frequencies).