Use Burg's algorithm to fit an AR(2) model to the six-point time series \{100, 10, 1, −1, −10, −100\}. Report what values you get for the Burg estimates \( \bar{\phi}_{1,2} \) and \( \bar{\phi}_{2,2} \) of the AR(2) coefficients and the corresponding estimate \( \bar{\sigma}^2 \) for the innovations variance. Assuming a sampling time of unity (i.e., \( \Delta t = 1 \)), state the corresponding estimated SDF in a reduced form (i.e, cosines rather than complex exponentials), and compute its value at frequencies \( f = 0, 1/4 \text{ and } 1/2 \) (alternatively, plot the SDF on a decibel scale versus frequency over a fine grid of frequencies, e.g., \( f_k = k/256 \) for \( k = 0, 1, \ldots, 128 \)).

The forward/backward least squares estimator of the coefficients \( \Phi \equiv [\phi_{1,2}, \phi_{2,2}]^T \) for an AR(2) process are defined as the values minimizing the sum of squares \( SS_{(fb)}(\Phi) \) stated in Equation (427). Show that the forward/backward least squares estimator \( \hat{\Phi}_{(fb)} \) satisfies the matrix equation

\[
\begin{bmatrix}
2 \sum_{t=2}^{N-1} X_t^2 \\
A \\
\sum_{t=3}^{N} X_t^2 + 2 \sum_{t=1}^{N-2} X_t^2
\end{bmatrix} \Phi = \begin{bmatrix}
2 \sum_{t=1}^{N-2} X_t X_{t+2}
\end{bmatrix}
\]

where you are to determine what \( A \) is. For the six-point time series, compute and report the two elements of \( \hat{\Phi}_{(fb)} \) and the corresponding estimate of the innovations variance, namely,

\[
\hat{\sigma}^2_{(fb)} = SS_{(fb)}(\hat{\Phi}_{(fb)})/(2[N - 2p])
\]

with \( N = 6 \) and \( p = 2 \). Do the estimated coefficients correspond to a stationary AR process? As for the Burg estimate, state the corresponding estimated SDF in a reduced form, and compute its values at frequencies \( f = 0, 1/4 \text{ and } 1/2 \) (alternatively, plot the estimated SDF in a manner similar to how you plotted the Burg estimate). How well do the Burg and forward/backward least squares estimates agree?