Exercise 3 of Assignment 8 (due 3/3/08)

Use Burg's algorithm to fit an AR(2) model to the six-point time series $\{100, 10, 1, -1, -10, -100\}$. Report what values you get for the Burg estimates $\bar{\phi}_{1,2}$ and $\bar{\phi}_{2,2}$ of the AR(2) coefficients and the corresponding estimate $\bar{\sigma}_2^2$ for the innovations variance. Assuming a sampling time of unity (i.e., $\Delta t = 1$), state the corresponding estimated SDF in a reduced form (i.e, cosines rather than complex exponentials), and compute its value at frequencies $f = 0$, $1/4$ and $1/2$ (alternatively, plot the SDF on a decibel scale versus frequency over a fine grid of frequencies, e.g., $f_k = k/256$ for $k = 0, 1, \ldots, 128$).

The forward/backward least squares estimator of the coefficients $\mathbf{\Phi} \equiv [\phi_{1,2}, \phi_{2,2}]^T$ for an AR(2) process are defined as the values minimizing the sum of squares $\mathrm{SS}_{(fb)}(\mathbf{\Phi})$ stated in Equation (427). Show that the forward/backward least squares estimator $\hat{\mathbf{\Phi}}_{(fb)}$ satisfies the matrix equation

$$\begin{bmatrix} 2\sum_{t=2}^{N-1} X_t^2 & A \\ A & \sum_{t=3}^{N} X_t^2 + \sum_{t=1}^{N-2} X_t^2 \end{bmatrix} \mathbf{\Phi} = \begin{bmatrix} A \\ 2\sum_{t=1}^{N-2} X_t X_{t+2} \end{bmatrix},$$

where you are to determine what A is. For the six-point time series, compute and report the two elements of $\hat{\mathbf{\Phi}}_{(fb)}$ and the corresponding estimate of the innovations variance, namely,

$$\hat{\sigma}_{(fb)}^2 = \mathrm{SS}_{(fb)}(\hat{\mathbf{\Phi}}_{(fb)})/(2[N-2p])$$

with $N = 6$ and $p = 2$. Do the estimated coefficients correspond to a stationary AR process? As for the Burg estimate, state the corresponding estimated SDF in a reduced form, and compute its values at frequencies $f = 0$, $1/4$ and $1/2$ (alternatively, plot the estimated SDF in a manner similar to how you plotted the Burg estimate). How well do the Burg and forward/backward least squares estimates agree?