Problem 1 - D-separation as undirected separation. Factorization

This is the same graph from Homework 2.

a. Write a topological ordering of the nodes in $V$.

b. Verify the following D-separation statements by

- constructing the respective ancestral graph
- moralizing the obtained ancestral graph
- checking undirected separation in the moral ancestral graph

$B \perp H \mid E$
$B \not\perp I \mid A,D,E$

c. Write the factored expression of a joint distribution $P_V$ over $V$ for which this graph is an I-map.

d. Verify that $B \perp I \mid EH$ in $P_V$ using marginalization in the factored form of $P_V$. 

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Problem 2 – A graph of tree-width 2

The graph $\mathcal{G}$ below has treewidth 2. The treewidth of a graph is one less than the size of the maximum clique in that graph.

![Graph $\mathcal{G}$](image)

a. Find an orientation for the graph $\mathcal{G}$, which produces no V-structures. Denote the resulting DAG by $\mathcal{G}'$.

b. Write the general factored form of a distribution $P$ for which the undirected graph is an I-map.

c. Write the general factored form of a distribution $P'$ of which the DAG you found in question a. is an I-map.

d. The two factorizations in b,c must be equal. Find a way to group the factors in $P'$ to obtain the factorization in $P$. The grouping may not be unique.

Using the grouping you found, show that the potentials of $P$ have a probabilistic interpretation. Since there will be many potentials, it is sufficient to find a probabilistic interpretation for one potential containing variable $A$ and for one containing the variable $E$.

e. Verify that $\mathcal{G}$ is chordal by the Tarjan elimination algorithm.

f. Construct a junction tree for the graph $\mathcal{G}$. Is this tree unique? List its separators (with multiplicities).