1. **(7 points)** For a continuous random variable with uniform density over the interval [40,100] find the median and interquartile range.

   The interval can be split into 4 equal width (equal probability) adjacent subintervals [40,55), [55,70), [70,85), [85,100] giving us median = 70 and IQR= 85 − 55 = 30.

2. **(8 points)** Suppose you are given the following ordered sample of size $n = 10$

   2.1, 2.4, 2.8, 3.2, 5.1, 6.5, 7.7, 10.2, 11.4, 12.0

   Give the plug-in estimates of all three quartiles.

   The three estimates are 2.8, $(5.1 + 6.5)/2 = 5.8$, and 10.2.

3. **(7 points)** Which six informative quantities does the R command `summary(x)` provide about a sample vector $x$?

   minimum, 1st quartile, mean, median, 3rd quartile and maximum.

4. **(9 points)** Explain briefly the distinction between sampled distribution and sampling distribution.

   The sampled distribution provides the sample $X_1, \ldots, X_n$ and the sampling distribution refers to the distribution of any statistics (say $\bar{X}_n$) computed from the sample.

5. **(6 points)** If a random variable $X$ has a $\mathcal{N}(\mu, \sigma^2)$ distribution, what is the distribution of $[(X − \mu) / \sigma]^2$?

   It is a chi-square distribution with one degree of freedom, since $Z = (X − \mu) / \sigma \sim \mathcal{N}(0,1)$.

6. **(6 points)** Give the mean and variance of $\bar{X}_{10}$ when it is computed for an i.i.d. random sample $X_1, \ldots, X_{10}$ from some distribution with mean $\mu = 1$ and standard deviation $\sigma = .2$. The final answers should consist of two decimal numbers.

   $E\bar{X}_{10} = \mu = 1$ and $\text{var} \bar{X}_{10} = \sigma^2/10 = (0.2)^2/10 = .004$.

7. **(5 points)** If `pnorm(1.5)` returns 0.9331928, what would `pnorm(-1.5)` return?

   `pnorm(-1.5)` would return $0.0668072 = 1 − 0.9331928$. 

8. **(12 points)** A task takes a random time \(X\) to complete, where \(E X = 2\) minutes and \(\text{var}(X) = \frac{1}{36}\) minutes\(^2\). What is the approximate chance that a person will complete 9 such tasks (one after the other) within 17 to 19 minutes, assuming that all 9 task times can be considered as independent random variables with the same distribution as \(X\). Express the answer using the appropriate R function with specific decimal number arguments. Give a good numerical estimate for the answer returned by this R expression.

\[
Y = X_1 + \ldots + X_9, \text{ then } EY = 9 \cdot 2 = 18 \text{ and } \text{var}(Y) = 9 \cdot \left(\frac{1}{36}\right) = \left(\frac{1}{2}\right)^2.
\]

Thus
\[
P(17 \leq Y \leq 19) = \text{pnorm}(19, 18, 0.5) - \text{pnorm}(17, 18, 0.5) = \text{pnorm}(2) - \text{pnorm}(-2) \approx 0.95
\]

9. **(3 points)** What does “i.i.d.” stand for?

independent identically distributed

10. **(6 points)** If \(X\) and \(Y\) are independent with means \(\mu_x = 20\) and \(\mu_y = 25\) and standard deviations \(\sigma_x = 1\) and \(\sigma_y = 2\), what are the mean and standard deviation of \(6X - 4Y\)? (Hint: the answer should consist of two clean integers)

\[
E(6X - 4Y) = 6EX - 4EY = 120 - 100 = 20 \text{ and } \text{var}(6X - 4Y) = 36 \cdot 1 + 16 \cdot 4 = 100 = 10^2, \text{ thus the standard deviation of } 6X - 4Y \text{ is } 10.
\]

11. **(8 points)** Explain the trade-off between the probability of type I error and the probability of type II error in hypothesis testing.

As one gets larger the other gets smaller. Increasing the rejection region \(R\) decreases the acceptance region \(R^c\) and vice versa.

12. **(8 points)** Should a significance probability of .99 cause us to reject the null hypothesis \(H_0\) when testing \(H_0\) against some alternative \(H_1\) at level \(\alpha = 0.05\)? Explain why or why not.

Since the signifiance probability 0.99 exceeds \(\alpha = 0.05\) we cannot reject \(H_0\) at this \(\alpha\).

13. **(5 points)** Fill in the question mark so that \(\text{pnorm}(?)\) produces the same as \(\text{pnorm}(3, \text{mean}=4, \text{sd}=1)\).

\[
?= (3 - 4)/1 = -1, \text{ the standardized value.}
\]

14. **(10 points)** What command in R would give you \(P(X \leq 15)\) when \(X\) denotes the number of times that you get a face value of 5 or 6 in 60 independent rolls of a fair die?

\(X \sim \text{Binomial}(60, 1/3). \text{ Thus } P(X \leq 15) = \text{pbinom}(15, 60, 1/3).\)