Closed Book! For the first five problems only circle the correct answer.

For problems 6, 7 and 8 show your work!

1) (5 points) For any $E \subset F^c$ we have that: $P(F^c \cap E^c) =$
   a) $1 - P(F) - P(E)$,   b) $1 - P(E)$,   c) 1,   d) 0,   e) $P(E) + P(F)$.

2) (5 points) $E^c \subset F$ implies:
   a) $F^c E = \emptyset$,   b) $E F = \emptyset$,   c) $F^c \subset E^c$,   d) $E \cup F = S$,   e) $F^c \cup E = S$.

3) (5 points) Using the 7 letters AEELSTT we can form ”words” such as SEATTLE or SELATTE etc. The number of distinct 7 letter words that can be made from these letters is:
   a) 770,   b) 840,   c) 910,   d) 1260,   e) 1540.

4) (5 points) For any events $A$ and $E$ we have that $P(A) =$
   a) $P(AE) + P(A^c E)$,   b) $P(E \vert A) / P(AE)$,   c) $P(AE) + P(AE^c)$,   d) $P(A \vert E) P(E)$.

5) (5 points) For any two events $A$ and $B$ we have $P(AB) =$
   a) $P(A \vert B)P(B \vert A)$   b) $P(A) + P(B) - P(A \cup B)$   c) $P(B \vert A)P(B)$,   d) $1 - P(A^c B^c)$.
6) **(15 points)** I have three dice. Two are normal and fair and the third has its six replaced by a one, but still shows all six faces with equal probability. I randomly hand you one of the three dice and you roll it three times.

a) What is the chance that you will see the sequence (1, 2, 3)?

b) Given that you see the sequence (1, 2, 3), what is the chance that I gave you the modified die?

7) **(10 points)** You shuffle a regular deck of 52 playing cards and deal a hand of 13 cards to player A. What is the chance that this hand will contain the queen of spades or the ace of hearts or both? (Be careful in counting!)
8) **(20 points)** An airplane engine after maintenance has functioned properly for 2 takeoffs but failed during the 3rd takeoff. From past experience it is known that an engine’s failure probability per takeoff is .7 whenever a certain seal is missing, but it is only .001, whenever the seal is present. Also, from experience we can assume the chance for noninstallation of this seal during the maintenance to be 2%.

a) What are the two chances of the above history \( F_1^c F_2^c F_3 \), i.e., (funct.,funct., failed), under either of the two seal conditions (“seal absent” = event \( A \) and “seal present” = event \( A^c \)), i.e., find \( P(F_1^c F_2^c F_3 | A) \) and \( P(F_1^c F_2^c F_3 | A^c) \). Assume independent trials for the takeoffs in either case?

b) What is the conditional chance for the seal to be missing given the observed history \( F_1^c F_2^c F_3 \)?