Back to the Future

Do recent developments tell us anything about the future of statistics?

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Outline

• Background
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- What recent developments?
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• What recent developments?
• Possible future directions?
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• What recent developments?
• Possible future directions?
• A few specific problems.
1. Background

Prediction is very difficult, especially when it involves the future. (Niels Bohr).
Some Boundaries and Restrictions:

• Focus here on:
  ◦ Statistical inference (theory and methods)
  ◦ Research problems
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- Other recent surveys, reviews, and reports:
  - Statistics in the 21st Century; Raftery, Tanner, and Wells. 22 vignettes (organized by George Casella) on “Theory and Methods”
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- Other recent surveys, reviews, and reports:
  - Longer on-line version of above: *Statistics: Challenges and Opportunities for the Twenty-First Century*.
  - *Statistics in the 21st Century*; Raftery, Tanner, and Wells.
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- Personal, subjective, and biased view, with lots ignored mostly due to personal ignorance.
Problems for the future (of Statistical Inference) mentioned by Kiefer, Savage, and Le Cam, 1967; Conference at University of Wisconsin on “The Future of Statistics”

- Problems mentioned by Kiefer:
  - Theory of nonparametric inference: testing and estimation.
  - Theory of nonparametric Bayes procedures. (Theory developing over last 10+ years: e.g. Le Cam lecture by van der Vaart at this meeting.)
  - Rates of convergence. (Götze, Bickel, van Zwet, Peter Hall, ... )
  - Nonparametric regression / curve estimation. (Hints of model selection, penalized estimation.)
• Problems, **Le Cam:**
  ◦ Alternatives to Bayesian statistics?
  ◦ Specification of stochastic structure?
  ◦ Stability of experiments? (Distance between experiments? Discrete versus continuous parameterizations?)
  ◦ Stability and relations to invariance?
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• Problems mentioned by Savage:
  ◦ Descriptive statistics (look at the data).
  ◦ Multi-parameter problems and nonparametric problems
  ◦ Weather forecasting?
  ◦ Medical diagnosis?
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    • “... select some subset of the variables if we have more than five or six ... ”
    • Change of perspective with computing power?
2. What recent developments?

The future ain’t what it used to be. (Yogi Berra)
Data: **Numbers of papers per year for several topics.**

- Topics explicitly mentioned at the 1967 Madison meeting
  - Nonparametric Bayes methods (Kiefer)
  - Robustness (Kiefer)
  - Subset regression (Mallows, Ball) → model selection
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- **Topics implicit or “hinted at” in discussion at the 1967 Madison meeting**
  - Alternatives to Bayes methods (Le Cam) → **empirical Bayes estimation**
  - Structure of stochastic models (Le Cam) → **graphical models**
  - Multiple comparisons / multiple testing (Savage) → **false discovery rate**
  - Multiparameter - nonparametric models (Savage) → **semiparametric models**
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  ◦ Bootstrap methods
  ◦ Markov chain monte carlo
  ◦ Empirical process theory / methods
  ◦ Lasso + regression
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• **Many other possible topics (not pursued here)**
  ◦ Hierarchical models
  ◦ Metaanalysis
  ◦ Nonparametric function estimation
  ◦ Causal inference
  ◦ Missing data
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Numbers of papers by year, Robustness
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WOS = ISI Web of Science (in Green)

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Numbers of papers by year, Robustness, all of MSN, CIS, WOS
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3. Possible future directions?

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  ◦ Networks ubiquitous: internet, social networks, citation networks, ...
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  ◦ Nested Laplace approximations: Rue, Martino, Chopin (2008)
• **Sparsity, sparse representations, compressed sensing**
  - Ingster (1993a, 1993b, 1997)
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• **More on model selection**
  ◦ Enormous qualitative changes in past 10 years:
  ◦ Changing perspectives: often no one “true” model.
  ◦ Replace with specified goals: prediction or variable/feature selection
  ◦ Often now based on “model averaging”, or “weighting”, or “aggregation” methods.
  ◦ Need for much more work on inference following model selection (e.g. H. Leeb, B. Pötscher)
4. Some specific problems (of special interest to me)

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• Shape restrictions and mixture models
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- \( \{\xi_i\}_{i=1}^N \), sampling indicators;
- \( \{\pi_i\}_{i=1}^N \), marginal inclusion probabilities;
- \( \{X_i\}_{i=1}^N \), population to be sampled.

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\mathbb{P}^\pi_N = N^{-1} \sum_{i=1}^N \frac{\xi_i}{\pi_i} \delta_{X_i} = \text{Horvitz - Thompson empirical measure},
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G^\pi_N = \sqrt{N}(\mathbb{P}^\pi_N - P)
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- In 1969 (time of the first moon landing) less than half of the current population of the U.S. had been born. (The national median age in the U.S. was 36.7 years in February 2009.)
A few references


