Semiparametric Regression Models for Panel Count Data: Comparing Two Estimators

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Outline

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• Cautions: identifiability issues and conditions for the theory
• Further work and Open Problems
1. Introduction: a Semiparametric Regression Model for Panel Count Data

I. Model for the Counting Process

- **A Mean structure:** \( E\{\bar{N}(t) | Z\} = e^{\theta'Z} \Lambda(t), \) 
  \( \Lambda \) monotone non-decreasing
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• Study estimators when the Poisson assumption B **fails**, but the conditional mean model given by A **holds**.
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  \[ 0 = T_{K,0} < T_{K,1} < \ldots < T_{K,K} \]
- $Z \sim H$ on $\mathbb{R}^d$
- No assumptions about $G$ or $H$
III. Data and Primary Goal:

- Data:

\[ X = (Z, K, T_K, \mathbb{N}(T_{K,1}), \ldots, \mathbb{N}(T_{K,K})) \]
\[ \equiv (Z, K, T_K, \mathbb{N}_K) \]

We observe \( X_1, \ldots, X_n \) i.i.d. as \( X \).
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- **Pictures!**
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• Pictures!

• Based on \( X_1, \ldots, X_n \) i.i.d. as \( X \), estimate \( (\theta, \Lambda) \)
Fig. 1: Counting process (green) and sampling process (red)
Figure 2. Counting process (green) and sampling process (red)
2. Maximum Pseudo-likelihood

and Maximum Likelihood Estimators

A. Maximum pseudo-likelihood.

- use the Poisson marginal distributions of $\mathbb{N}$,

$$
P(\mathbb{N}(t) = k | Z) = \frac{\Lambda(t | Z)^k}{k!} \exp(-\Lambda(t | Z))
$$

and ignore dependence between $\mathbb{N}(t_1)$ and $\mathbb{N}(t_2)$ to obtain the pseudo-likelihood

$$
lp_n(\theta, \Lambda) = \sum_{i=1}^{n} \sum_{j=1}^{K_i} \left\{ \mathbb{N}^{(i)}(T^{(i)}_{K_i,j}) \log \Lambda(T^{(i)}_{K_i,j}) + \mathbb{N}^{(i)}(T^{(i)}_{K_i,j}) \theta' Z_i - e^{\theta' Z_i} \Lambda(T^{(i)}_{K_i,j}) \right\}.
$$
Then
\[
(\hat{\theta}_{ps}^n, \hat{\Lambda}_{ps}^n) \equiv \arg\max_{\theta, \Lambda} l_n^{ps}(\theta, \Lambda).
\]

Implement in two steps:
\[
\hat{\Lambda}_{ps}^n (\cdot, \theta) \equiv \arg\max_{\Lambda} l_n^{ps}(\theta, \Lambda),
\]
and define
\[
l_n^{ps, profile}(\theta) \equiv l_n^{ps}(\theta, \hat{\Lambda}_{ps}^n (\cdot, \theta)).
\]
Then
\[
\hat{\theta}_{ps}^n = \arg\max_{\theta} l_n^{ps, profile}(\theta),
\]
and
\[
\hat{\Lambda}_{ps}^n = \hat{\Lambda}_{ps}^n (\cdot, \hat{\theta}_{ps}^n).
\]
Let $t_1 < \ldots < t_m$ denote the ordered distinct observation time points in the collection of all observations times, 
$\{T_{K_i,j}^{(i)}, j = 1, \ldots, K_i, i = 1, \ldots, n\}$, and set 

\[
\begin{align*}
    w_l &= \sum_{i=1}^{n} \sum_{j=1}^{K_i} 1_{[T_{K_i,j}^{(i)} = t_l]}, \\
    \overline{N}_l &= \frac{1}{w_l} \sum_{i=1}^{n} \sum_{j=1}^{K_i} \overline{N}_{K_i,j}^{(i)} 1_{[T_{K_i,j}^{(i)} = t_l]}, \\
    \overline{A}_l(\theta, Z) &= \frac{1}{w_l} \sum_{i=1}^{n} \sum_{j=1}^{K_i} \exp(\theta' Z^{(i)}) 1_{[T_{K_i,j}^{(i)} = t_l]}.
\end{align*}
\]

Then the cumulative sum diagram is given by 

\[
\{(\sum_{l \leq i} w_l \overline{A}_l(\theta, Z), \sum_{l \leq i} w_l \overline{N}_l)\}_{i=1}^{m}
\]
\[
\Lambda_{n}^{ps}(\cdot, \theta) = \text{left-derivative of } \text{Greatest Convex Minorant}
\]
\[
\text{of } \left\{ \left( \sum_{l \leq i} w_{l} A_{l}(\theta, Z), \sum_{l \leq i} w_{l} N_{l} \right) \right\}_{i=1}^{m}
\]
\[
= \max_{i \leq l} \min_{j \geq l} \frac{\sum_{i \leq p} w_{p} N_{p}}{\sum_{i \leq p} w_{p} A_{p}(\theta, Z)} \text{ at } t_{l},
\]

which is easy to compute.
B. Maximum likelihood: use the independence of the increments
\( \Delta N(s, t] \equiv N(t) - N(s) \), and the Poisson distribution of these
increments of \( N \),

\[
P(\Delta N(s, t] = k|Z) = \frac{[\Delta \Lambda((s, t]|Z)]^k}{k!} \exp(-\Delta \Lambda((s, t]|Z))
\]

to obtain the log-likelihood:

\[
l_n(\theta, \Lambda) = \sum_{i=1}^{n} \sum_{j=1}^{K_i} \left\{ \Delta N^{(i)}((T^{(i)}_{K_i,j-1}, T^{(i)}_{K_i,j}) \cdot \log \Delta \Lambda((T^{(i)}_{K_i,j-1}, T^{(i)}_{K_i,j}))
\right.
\]

\[
+ \Delta N^{(i)}((T^{(i)}_{K_i,j-1}, T^{(i)}_{K_i,j}))\theta' Z_i
\]

\[
- \exp(\theta' Z_i)\Lambda((T^{(i)}_{K_i,j-1}, T^{(i)}_{K_i,j})) \right\}
\]
Then the MLE is \((\hat{\theta}_n, \hat{\Lambda}_n) \equiv \arg\max_{\theta, \Lambda} l_n(\theta, \Lambda)\).

Implement this maximization in two steps (profile likelihood):

\[ \hat{\Lambda}_n(\cdot, \theta) \equiv \arg\max_{\Lambda} l_n(\theta, \Lambda), \]

and define \(l_n^{\text{profile}}(\theta) \equiv l_n(\theta, \hat{\Lambda}_n(\cdot, \theta))\). Then

\[ \hat{\theta}_n = \arg\max_{\theta} l_n^{\text{profile}}(\theta), \quad \hat{\Lambda}_n = \hat{\Lambda}_n(\cdot, \hat{\theta}_n). \]

Computation of the (profile) “estimator” \(\hat{\Lambda}_n(\cdot, \theta)\) is hard, but possible: iterative convex minorant algorithm.
3. Properties of the Estimators
when the Poisson Assumption Fails

Theorem 1. If assumption A holds, then (under further integrability, boundedness, and identifiability hypotheses):

$$\sqrt{n}(\hat{\theta}_n - \theta_0) \xrightarrow{d} Z \sim N_d \left(0, A^{-1} B (A^{-1})' \right),$$

and

$$\sqrt{n}(\hat{\theta}_{n}^{ps} - \theta_0) \xrightarrow{d} Z^{ps} \sim N_d \left(0, (A^{ps})^{-1} B^{ps} ((A^{ps})^{-1})' \right)$$

where $A, B, A^{ps},$ and $B^{ps}$ are given by:
\[ B = E m^*(\theta_0, \Lambda_0; X) \otimes^2 \]

\[ = E \left\{ \sum_{j,j' = 1}^K C_{j,j'}(Z) \left[ Z - \frac{E (Z e^{\theta'_0 Z} | K, T_{K,j,j'})}{E (e^{\theta'_0 Z} | K, T_{K,j,j'})} \right]^2 \right\} \]

\[ A = E \left\{ \sum_{j = 1}^K \Delta \Lambda_0 K_j e^{\theta'_0 Z} \left[ Z - \frac{E (Z e^{\theta'_0 Z} | K, T_{K,j,j-1})}{E (e^{\theta'_0 Z} | K, T_{K,j,j-1})} \right]^2 \right\} , \]

\[ C_{j,j'}(Z) = \text{Cov} [\Delta N_{K,j}, \Delta N_{K,j'} | Z, K, T_K] . \]
\[
B^{ps} = E m^{*ps}(\theta_0, \Lambda_0; X)^{\otimes 2}
\]
\[
= E \left\{ \sum_{j,j'=1}^{K} C_{j,j'}^{ps}(Z) \left[ Z - \frac{E (Ze^{\theta_0}Z|K,T_{K,j})}{E (e^{\theta_0}Z|K,T_{K,j})} \right] \right\}^{\otimes 2},
\]
\[
A^{ps} = E \left\{ \sum_{j=1}^{K} \Lambda_0 K_j e^{\theta_0}Z \left[ Z - \frac{E (Ze^{\theta_0}Z|K,T_{K,j})}{E (e^{\theta_0}Z|K,T_{K,j})} \right] \right\}^{\otimes 2},
\]
\[
C_{j,j'}^{ps}(Z) = \text{Cov} [N_{K,j}, N_{K,j'}|Z, K, T_{K,j,j'}],
\]

If the Poisson process assumption B holds,
\[
A = B = I(\theta),
\]
and \( \hat{\theta}_n \) is (asymptotically) \textbf{efficient}. 
4. Efficiency comparisons:

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  - \((T_K | K) \sim \) order statistics of \( K \) i.i.d. \( U[0, M] \) rv’s
  - \( K \sim \) one of:
    - (a) Degenerate at \( k_0 \)
    - (b) (Shifted by 1) Poisson(\( \gamma \))
    - (c) Discrete zeta(\( \alpha \)); \( P(K = k) = \frac{1/k^\alpha}{\zeta(\alpha)}, \zeta(\alpha) = \sum_{j=1}^{\infty} j^{-\alpha} \)
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  - $K \sim$ one of:
    (a) Degenerate at $k_0$
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\[
ARE(pseudo, mle) = \frac{[E(K/2)]^2}{E \left\{ \frac{K}{K+1} \right\} E \left\{ \frac{K(2K+1)}{6} \right\}}
\]
Case (a):

\[ ARE(pseudo, mle)(k_0) = \frac{3}{4} \frac{k_0 + 1}{k_0 + 1/2} \]
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• Case (b):

\[ ARE(pseudo, mle)(\gamma) = \frac{3}{2} \frac{(\gamma + 1)^2}{(2\gamma^2 + 7\gamma + 3)E_\gamma \left\{ \frac{K}{K+1} \right\}}. \]
• **Case (a):**

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\]

• **Case (c):**

\[
ARE(pseudo, \text{mle})(\alpha) = \frac{3}{2} \frac{\zeta(\alpha - 1)}{\{2\zeta(\alpha - 2) + \zeta(\alpha - 1)\} E_{\alpha} \left\{ \frac{K}{K+1} \right\}}.
\]
Figure 3. Relative efficiency, scenario 1(a): $K$ degenerate at $k_0$ as a function of $k_0$. 

Semi-parametric Regression Models for Panel Count Data: Comparing Two Estimators – p. 20/32
Figure 4. Relative efficiency, scenario 1(b): $K$ shifted Poisson as a function of $\gamma$
Figure 5. Relative efficiency, scenario 1(c): $K$ discrete zeta
Scenario 2: Suppose that:

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- $K \sim$ one of:
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  - (c) Discrete zeta($\alpha$); $P(K = k) = \frac{1/k^\alpha}{\zeta(\alpha)}$, $\zeta(\alpha) = \sum_{j=1}^{\infty} j^{-\alpha}$
\[
\text{ARE}(\text{pseudo, mle})(\text{NegBin}) = \frac{\left(1 + a \frac{E\left(\frac{K}{K+2}\right)}{E\left(\frac{K}{K+1}\right)}\right)}{\left(1 + a \frac{E\left(\frac{K(3K+1)}{12}}{E\left(\frac{K(2K+1)}{6}\right)}\right)} \cdot \text{ARE}(\text{pseudo, mle})(\text{Poisson}).
\]

where \( a \equiv q/p = \lambda M/\gamma \).
Figure 6. Relative efficiency, scenario 2, as a function of $q/p$
Figure 7. Relative efficiency, scenario 2, as a function of $\kappa$
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- Example: suppose that
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  ○ $\Lambda(t) = t$, $\beta = 1$
  
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  ○ Then $\Lambda_0(T)e^{\beta_0 Z} = T^2 = \Lambda(T)e^{\beta Z}$ almost surely and the model is not identifiable.
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  ◦ Then $\Lambda_0(T)e^{\beta_0 Z} = T^2 = \Lambda(T)e^{\beta Z}$ almost surely and the model is not identifiable.

• Conditions needed to be able estimate both $\Lambda$ and $\theta$!
• Some measures:

\[ \nu_1(B \times C) = \int_C \sum_{k=1}^{\infty} P(K = k \mid Z = z) \sum_{j=1}^{k} P(T_{k,j} \in B \mid K = k, Z = z) dH(z), \]

\[ \mu_1(B) = \nu_1(B \times \mathbb{R}^d) \]

\[ \nu_2(B_1 \times B_2 \times C) = \int_C \sum_{k=1}^{\infty} P(K = k \mid Z = z) \cdot \sum_{j=1}^{k} P(T_{k,j-1} \in B_1, T_{k,j} \in B_2 \mid K = k, Z = z) dH(z), \]

\[ \mu_2(B_1 \times B_2) = \nu_2(B_1 \times B_2 \times \mathbb{R}^d) \]
• $C_2^{ps}: \mu_1 \times H << \nu_1$ (needed for identifiability - consistency of Poisson-based pseudo MLE)
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• $C_2: \mu_2 \times H << \nu_2$ (needed for identifiability - consistency of Poisson-based MLE)
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• See **Technical Report 488, UW Department of Statistics**
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• Weaker hypotheses needed!
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