hw-B Another example of a density function is the uniform density function, defined as

$f(x) = \frac{1}{d-a}$ for $a < x < d$, and ϕ otherwise.

a) Show that this $f(x)$ is a density function.

To be a density, $f(x)$ must satisfy 2 conditions:

$f(x) \geq 0$ and $\underbrace{\int_{-\infty}^{\infty} f(x)\,dx}$ $= 1.$

↑

This is true because $d > a$

$$= \int_a^d \frac{1}{d-a}\,dx$$

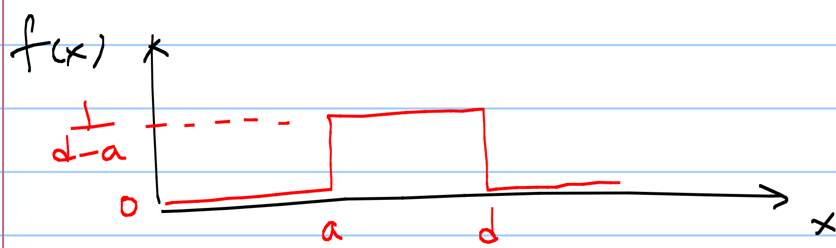
$$= \frac{1}{d-a}\int_a^d dx = \frac{d-a}{d-a} = 1 \checkmark$$

b) What proportion of times will x be between b and c, where $a < b < c < d$?

$$\int_b^c \frac{1}{d-a}\,dx = \frac{1}{d-a}\int_b^c dx = \boxed{\frac{c-b}{d-a}}.$$

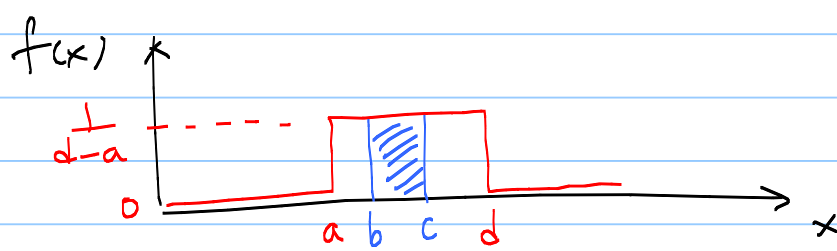
It's best to draw pictures of distributions.

The uniform distr., as defined above, looks like this:



You can then see easily that the area under f(x) is in fact 1.

You can also see why the prop between b and c is what it is:



area = height × width

$= \frac{1}{d-a} \cdot (c-b)$

= same as the $\int$.