

hw-d

Suppose the density function for x is given by the normal distribution with parameters μ, σ .

I.e. $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $f(z)$

a) compute the density function for $z = \frac{x-\mu}{\sigma}$.

Hint: start with $\int_{-\infty}^{\infty} f(x) dx = 1$ and derive $\int_{-\infty}^{\infty} f(z) dz = 1$.

$$\int_{-\infty}^{\infty} f(x) dx = 1 \implies \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1$$

$$z = \frac{x-\mu}{\sigma} \implies dz = \frac{1}{\sigma} dx$$

$$\frac{1}{\sqrt{2\pi}} \int e^{-\frac{1}{2}z^2} dz = 1 \implies f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

b) From the form of $f(z)$, read off its μ and σ parameters.

$$\mu = 0, \sigma = 1.$$