

hw-F

Suppose you have computed the sample mean of a sample of size n . Call it $\bar{x}_n = \frac{1}{n} \sum_{i=1}^n x_i$. Now, suppose a new observation is made; call it x_{n+1} . What is the relationship between \bar{x}_n and the "new" mean of all $(n+1)$ observations, denoted \bar{x}_{n+1} ? Show work.

The mean of the $(n+1)$ observations is

$$\bar{x}_{n+1} = \frac{1}{n+1} \sum_{i=1}^{n+1} x_i$$

$$= \frac{1}{n+1} \left[\underbrace{x_1 + x_2 + \dots + x_n}_{\sum_{i=1}^n x_i} + x_{n+1} \right]$$

$$= \frac{1}{n+1} \underbrace{\sum_{i=1}^n x_i}_{n \bar{x}} + \frac{x_{n+1}}{n+1}$$

$$\bar{x}_{n+1} = \frac{n}{n+1} \bar{x} + \frac{x_{n+1}}{n+1}$$

Note:
Good programmers often use this formula to "update" the mean, without computing it each time!