hw_G Here are some data:

115.8, 115.2, 114.6, 115.9, 116.4

a) Compute the sample standard deviation using the defining formula.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{5}(115.8 + \cdots) = 115.58.$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2} = \sqrt{\frac{1}{4}(115.8-115.58)^2 + \cdots} = \underline{\underline{0.6942}}$$

b) compute it using the computational formula.

$$s = \sqrt{\frac{n}{n-1}\left(\overline{x^2} - \bar{x}^2\right)}$$

$$(115.58)^2 = 13358.74$$

$$\frac{1}{n}\sum_i x_i^2 = \frac{1}{5}\left[115.8^2 + \cdots\right] = 13359.12$$

$$s = \sqrt{\frac{5}{4}\cdot(0.3856)} = \underline{\underline{0.6942}}$$

c) Subtract 100 from each value, and compute the sample std. dev. using either formula. Compare with a or b.

New x : 15.8, 15.2, 14.6, 15.9, 16.4

$$\bar{x} = 15.58$$

$$s = \sqrt{\frac{1}{4}(15.8-15.58)^2 + \cdots} = \underline{\underline{0.6942}}$$

Same!

d) Now generalize to any data, x_i, $i = 1, 2, \ldots, n$.
I.e. show that $x_i \rightarrow x_i - c$, where c is a constant, does not affect the std. dev.

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

First, let's find out how $\bar{x}$ changes as

$x_i \longrightarrow x_i - c$:

$$\bar{x} \longrightarrow \frac{1}{n} \sum_{i=1}^{n} (x_i - c) = \underbrace{\frac{1}{n} \sum_{i=1}^{n} x_i}_{\bar{x}} - \frac{c}{\cancel{n}} \sum_{i=1}^{n} 1$$

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So, $\bar{x} \longrightarrow \bar{x} - c$.

Now, how does s^2 change?

$$s^2 \longrightarrow \frac{1}{n-1} \sum_{i=1}^{n} \left[(x_i - \cancel{c}) - (\bar{x} - \cancel{c}) \right]^2$$

$$= \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= s^2$$

So, $s^2 \longrightarrow s^2$, i.e. $\underline{\underline{s \longrightarrow s}}$