

One way:

$$r = \frac{1}{n-1} \sum_i \left(\frac{x_i - \bar{x}}{S_x} \right) \left(\frac{y_i - \bar{y}}{S_y} \right)$$

$$r = \frac{1}{S_x S_y} \frac{1}{n-1} \sum_i (x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{\sqrt{\cancel{n-1} \sum (x_i - \bar{x})^2}} \cdot \frac{1}{\sqrt{\cancel{n-1} \sum (y_i - \bar{y})^2}} \cdot \cancel{n-1} \sum (x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

Since S_x and S_y have no i index, they are constants.

Defn of S_x, S_y .

That's the end of that hw, but if you also want to see how the 2nd eqn on p. 108 enters the game, check out the page, below.

Expand the products in the last eqn. above:

$$\therefore r = \frac{\sum (x_i y_i) - \sum \bar{x} y_i - \sum \bar{y} x_i + \sum \bar{x} \bar{y}}{\sqrt{\sum x_i^2 - 2 \sum \bar{x} x_i + \sum (\bar{x})^2} \sqrt{\text{same for } y}}$$

$$= \frac{\sum x_i y_i - \bar{x} \sum y_i - \bar{y} \sum x_i + \bar{x} \bar{y} \sum 1}{\sqrt{\sum x_i^2 - 2 \bar{x} \sum x_i + (\bar{x})^2 \sum 1} \sqrt{\dots}}$$

$$= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sqrt{\sum x_i^2 - n (\bar{x})^2} \sqrt{\dots}} = \frac{\sum x_i y_i - \frac{1}{n} (\sum x_i) (\sum y_i)}{\sqrt{\sum x_i^2 - \frac{1}{n} (\sum x_i)^2} \sqrt{\dots}}$$

$$= \underline{S_{xy} / (S_{xx} S_{yy})}$$