

$$\text{MSE}(\alpha, \beta) = \frac{1}{n} \sum_{i=1}^n (y_i - \alpha - \beta x_i)^2$$

$$\frac{\partial}{\partial \alpha} \text{MSE}(\alpha, \beta) = \frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \alpha} (y_i - \alpha - \beta x_i)^2$$

} chain rule.

$$= \frac{2}{n} \sum_{i=1}^n (y_i - \alpha - \beta x_i) \cdot \frac{\partial}{\partial \alpha} (-\alpha)$$

-1

$$= \frac{2}{n} \left( \underbrace{\sum_{i=1}^n y_i}_{n\bar{y}} - \alpha \underbrace{\sum_{i=1}^n 1}_n - \beta \underbrace{\sum_{i=1}^n x_i}_{n\bar{x}} \right)$$

$$= 2 (\bar{y} - \alpha - \beta \bar{x})$$

$$\frac{\partial}{\partial \alpha} \text{MSE} \Big|_{\hat{\alpha}, \hat{\beta}} = 0 \quad \Rightarrow \quad \underline{\bar{y} - \hat{\alpha} - \hat{\beta} \bar{x} = 0}$$