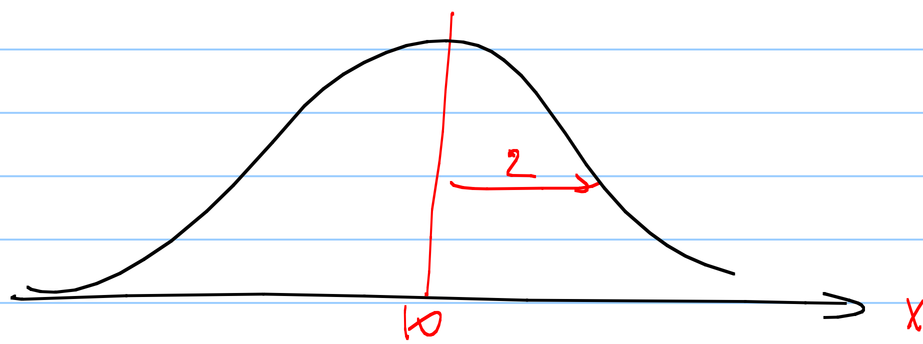
As we keep saying the formulas we've developed have the pop. parameters on the right-hand side; these are quantities we don't know, and want to infer.

Here is one intuitive way of handling this situation: "proof by contradiction".

$\boxed{\text{Assume}}$ the pop. distribution of height (x) is:

$$N(\mu = 10, \sigma = 2).$$



What's the prob. that a random person will be taller than $X = 12$?

$$P(X > 12) = P\left(\frac{X-\mu}{\sigma} > \frac{12-10}{2}\right)$$

$$= P(Z > 1) = 0.16$$

Now, CLT tells us that $\bar{x}$'s are distributed as $\bar{x} \in N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$

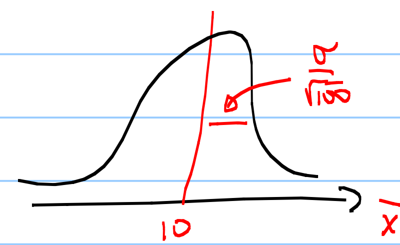
Q What's the prob. that a <u>random sample</u> of $n = 100$ will have a <u>mean</u> exceeding $\bar{x} = 12$?

$$P(\bar{x} > 12) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{12-10}{2/\sqrt{100}}\right)$$

$$= P(z > 10) \simeq 0$$

any old $\bar{x}$, i.e. $\bar{x}$ = r.v.

obs sample mean, $\bar{x}_{obs}$



observed sample mean.

Q If <u>the sample yields</u> $\bar{x} = 12$ (or larger), "$\bar{x}_{obs}$"

is our assumption (that the distribution of heights in the population is $N(\mu=10, \sigma=2)$) reasonable? No!

How about $N(\mu \le 10, \sigma=2)$? No!

∴ Either $\mu \le 10$ is false, or $\sigma = 2$ is false, or both.

Conclusion: If $\bar{x} = 12$, then we should feel compelled to reject the assumption that the pop. is normal with $\mu \le 10$, $\sigma = 2$. More in Ch8!

Summary: we assumed the population, and computed the prob. of a sample statistic exceeding the one observed from data.

R.V. $\bar{X}_{observed}$

I.e. assume pop $= N(\mu=10, \sigma=2)$. Then, $p(\bar{X} \geq 12) \simeq 0$.

The single sample mean.

Conclusion: pop. is probably not $N(\mu \leq 10, \sigma=2)$

That's one way of "testing" the population.

E.g. Reject the claim that pop. is $N(\mu \leq 10, \sigma=2)$

We'll return to this approach in Ch. 8.