Lecture 4 (Ch 1)

Last time: hists ≠ dists. 1) Exp. dist, 2) poisson dist
In hw, you'll see the uniform dist. Now, a bit more:

3) Binomial, \( x = \text{discrete} \)

We'll derive its mass function, next time, but it's:

\[
p(x) = \begin{cases} 
\frac{n!}{x! (n-x)!} \theta^x (1-\theta)^{n-x}, & x = 0, 1, \ldots, n \\
0, & \text{otherwise}
\end{cases}
\]

Note it is a mass function: \( p(x) \geq 0 \), \( \sum_{x=0}^{n} p(x) = 1 \)

It has parameters/meaning: \( n, \theta \) \([n=\text{integers}, 0<\theta<1]\)

Depending on the value of the params, it can look like these:

In lab you'll see how these look for different \( \theta \) values.

E.g. 
- # of Heads out of \( n \) tosses.
- # of defective gates on a chip with \( n \) gates.
- # of girls in a sample of size \( n \).

Etc.
4) Normal/Gaussian, \( x = \text{const} \).

E.g. \[ f(x) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2} \]

- \( \mu \): measure of "location" or centrality.
- \( \sigma \): measure of "spread".

Important: Resist the temptation to call \( \mu \) and \( \sigma \) "mean" and "standard deviation", at least for now. Otherwise, you'll get very confused. They are simply parameters of the distribution.
As I said, given a distr., we can compute things called mean, median, mode, etc. The proportion of times $x$ will be between 2 values:

- For binomial $(n, p)$:
  \[ \sum_{k=a}^{b} \binom{n}{k} p^k (1-p)^{n-k} \]

- For Normal $(\mu, \sigma)$:
  \[ \int_{a}^{b} \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2} \, dx \]

- For std. Normal:
  \[ \frac{1}{\sqrt{2\pi}} \int_{a}^{b} e^{-\frac{1}{2} x^2} \, dx \]

Note: Std. Normal = Normal $(\mu=0, \sigma=1)$.

Unfortunately, integrals of this type can be done only numerically. Their values are tabulated in Table I.

Eg.

\[ \int_{-2.23}^{2.23} \text{for} \, dx = 0.0129 \]

Note: Std. Normal is symmetric.

In 390, use Table I, not computers/calculators, except for problems that say “By R.”
But Table I gives only “left areas.” So, we have to be tricky/smart about finding areas between 2 numbers:

$$\text{prop}(a < x < b) = \text{area between } a \text{ & } b =$$

$$= \text{prop}(x < b) \ominus \text{prop}(x < a)$$

Both of these can be obtained from Table I.

**Example:** What is the area under the std. Normal between -1 and +1?

$$= 0.8413 - 0.1587 = 0.68$$

“famous” 68%.

**Example:** How about between -2.1 and 0?

$$= 0.5 - 0.179 = 0.321$$

0.4821
Now, we know how to find area/prop. under std. normal.

How do we handle $\mathcal{N}(\mu, \sigma)$?

It would be impossible to tabulate values for every value of the 2 parameters, $\mu, \sigma$. Need one more trick!

The trick is to "standardize" (i.e. change variables):

$$X \rightarrow Z = \frac{X - \mu}{\sigma} \quad (Z - score)$$

So, to compute area between 2 values:

$$\text{prop}(a < X < b) = \text{prop} \left( \frac{a - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{b - \mu}{\sigma} \right)$$

Either way (algebraically or graphically) you can obtain the value of each term from Table 1.

Recall that Table 1 gives areas for Z-scores.
Example: What's the area between -2 and +2 for a normal curve with $\mu = 4$, $\sigma = 3$?

\[ z = \frac{2 - 4}{3} = \frac{2 - 4}{3} = -\frac{2}{3} = -0.67 \]

\[ 0.2544 - 0.0228 = 0.2316 \]

Example: How about within 1 of $\mu$?

\[ z = \frac{1 - 4}{3} = \frac{1 - 4}{3} = -1 \]

\[ 0.8413 - 0.1587 = 0.68 \]

Remember, 68%.

It's a useful thing to know, but don't overuse it!
Suppose the density function for $x$ is given by the Normal dist. with parameters $\mu$, $\sigma$. I.e.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2}$$

\[ a) \] Compute the density function $f(z)$, for $z = \frac{x-\mu}{\sigma}$.

**Hint:** $f(z)$ must satisfy $\int_{-\infty}^{\infty} f(z) \, dz = 1$.

So, start with $\int_{-\infty}^{\infty} f(x) \, dx = 1$, with $f(x)$ as above, and massage the expression until it becomes $\int_{-\infty}^{\infty} f(z) \, dz = 1$.

Then $f(z) = \ldots$.

**Note:** It is not necessary to perform any integrals.

\[ b) \] From the form of $f(z)$, read off its $\mu$ and $\sigma$ parameters. I.e. What are their values?