Name: ________________________

ID: ________________________

Quiz section or time: ________________________

4+13

Stat/Math 390, Autumn, Test 1 - Oct. 10, 2008; Marzban

Open everything, closed messaging/discussion

Check front page and back page.

Multiple-choice: mark answers on these pages.

The rest: SHOW your answer and your WORK, on these pages.

Points NO CREDIT FOR CORRECT ANSWER WITHOUT EXPLANATION.

1 **1.** Let x be the number of correct answers on a test with 4 multiple-choice questions, each with 3 possible answers. The most appropriate distribution for describing this variable is

a) Normal b) Binomial c) Poisson d) Exponential

↑ continuous ↑ for $n \to \infty$, $\pi \to 0$ ↑ continuous

1 **2.** For a given data set, is it true that a value of x three standard deviations from the mean is farther away from the sample mean than a value of x two standard deviations away from the sample mean? Mark the most appropriate answer.

a) Always yes. b) Yes, but only if data is Normal. c) Yes, but depending on the data.

1 p.25 **3.** For a continuous variable, x, the proportion of times that a specific value, x_0, can occur is equal to the value of the density function at x_0.

a) True. b) True, but only for a Normal distribution. c) Definitely false.

The prop $(x = x_0) = 0$, because the area over a point is zero.

1 1.58 **4.** A paper reports that the number of defective transistors across a square centimeter of an integrated circuit follows the Poisson distribution with $\lambda = 100$. But you are interested in the distribution of defective transistors across half of that area. What value of λ should you use?

a) 100 b) 50 c) 200 d) Insufficient information provided.

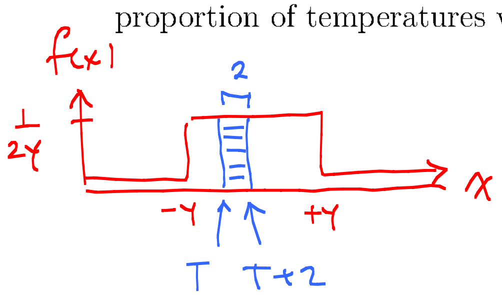
2 example 1.12 **5.** Compute the median of an exponential distribution with parameter λ.

$$\int_0^c \lambda e^{-\lambda x}\, dx = \frac{1}{2} \Rightarrow \left.\frac{\lambda e^{-\lambda x}}{-\lambda}\right|_0^c = \frac{1}{2}$$

$$e^{-\lambda c} - 1 = -\frac{1}{2} \Rightarrow e^{-\lambda c} = \frac{1}{2} \Rightarrow \lambda c = \ln(2)$$

$$\boxed{c = \ln(2)/\lambda}$$

2 1.20c **6.** Suppose that the reaction temperature x (in C) in a certain process has a uniform distribution between -y (C) and +y (C). For any temperature T satisfying $-y < T < T + 2 < y$, what long-run proportion of temperatures will be between T and $T + 2$?



prop $(T < x < T+2)$

= area of blue region

$= 2\left(\frac{1}{2y}\right)$

$= \boxed{\frac{1}{y}}$

3 2.65 or 1.73 **7.** Wine is considered to be properly handled if the storage temperature is within 1 standard deviation of the mean. If these wine bottles are shipped in batches of 20, in the long run what proportion of the shipments will have at least 19 bottles that are properly handled? Give a numerical answer, and show work. Assume storage temp. is Normally distributed.

$X = \#$ of properly handled bottles out of 20.

$$\text{prop}(X \geq 19) = p(X=19) + p(X=20) = \binom{20}{19}\pi^{19}(1-\pi)^1 + \binom{20}{20}\pi^{20}(1-\pi)^0$$

$.0042 + .0004 = \boxed{.0046}$

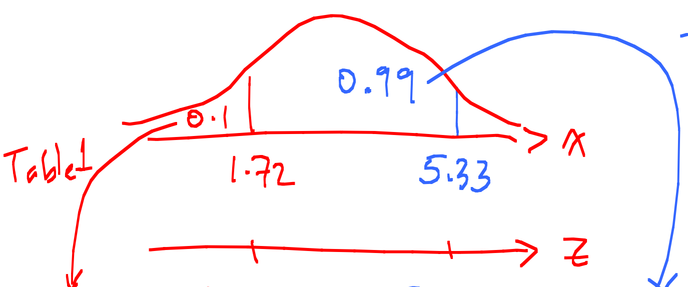
$$= 20\pi^{19} - 20\pi^{20} + \pi^{20} = 20\pi^{19} - 19\pi^{20}.$$

π = prop. of properly handled bottles

= area under Normal within 1 std. dev. of mean $= 0.68$

$$\therefore \text{prop}(X \geq 19) = 20(0.68)^{19} - 19(0.68)^{20} = 0.0131 - .0085 = \boxed{.0046}$$

3 2.68 **8.** Let x denote the maximum physical stress that a unit of a certain product encounters during its lifetime. Suppose that x is normally distributed with the 99th percentile = 5.33 and the 10th percentile is 1.72. What are the values of the μ and σ of the distribution? Give numerical answers, and show work.



$$-1.28 = \frac{1.72-\mu}{\sigma} \qquad \frac{5.33-\mu}{\sigma} = 2.33$$

$\therefore$ $5.33 - \mu = 2.33\sigma$ ⎫ 2 eqns,
$1.72 - \mu = -1.28\sigma$ ⎭ 2 unknowns

$(-)$ $3.61 = 3.61\sigma \implies \boxed{\sigma = 1}$

$5.33 - \mu = 2.33(1) \implies \boxed{\mu = 3}$

3 2.61 **9.** Consider a sample $x_i, i = 1, 2, ..., n$ with mean $\overline{x}$, and standard deviation s, and let $z_i = (x_i - \overline{x})/s$. What are the mean and standard deviation of the z_i's? Show work.

$$\underbrace{\text{mean of } z}_{\overline{z}} = \frac{1}{n}\sum_{i=1}^{n} z_i = \frac{1}{n}\sum_i\left(\frac{x_i - \overline{x}}{s}\right) = \frac{1}{ns}\underbrace{\sum_i (x_i - \overline{x})} = \boxed{0}$$

$\sum_i x_i - \overline{x}\sum_i 1$ ⎫ p. 4
$n\overline{x} - \overline{x}n$ ⎭ lect. 5

$$(\text{std. dev. of } z)^2 = \frac{1}{n-1}\sum_i (z_i - \overline{z})^2 = \frac{1}{n-1}\sum_i\left(\frac{x_i - \overline{x}}{s}\right)^2$$

$$= \frac{1}{s^2}\underbrace{\frac{1}{n-1}\sum_i (x_i - \overline{x})^2}_{= s^2} = \frac{s^2}{s^2} = \boxed{1}$$

$\therefore$ $\overline{z} = 0$, $s_z = 1$ for any data