

Quiz 2

March 22

Your section: _____ Print your name: _____

Sign your name: _____

This is an open book exam. The total number of points is 30, but the maximum you can score is 25. You are not allowed to pass any material (such as books, notes, or calculators) to each other. The quiz consists of THREE problems and you have 30 minutes. Show as much work as possible. Good luck!

Problem 1. A Stat 220 class has 80 male students and 80 female students. A student is picked at random.

- (a) What is the chance that the student picked is female ?

I now tell you that the chance of picking a male sophomore student, at random, from this class is $1/4$. I also tell you that the class has 60 sophomore students.

- (b) What is the chance of picking a sophomore student at random ? What is the (conditional) chance of picking a sophomore student given that they are female ?
- (c) What is the (conditional) chance of picking a male student given that they are sophomore ?

(2+5+3 = 10 points)

Solution:

- (a) Total number of students = $80+80 = 160$. Out of these 80 are female. Therefore the chance that a student picked at random is female is simply $80/160 = 1/2 = 0.5$.
- (b) Since the chance of picking a male sophomore student at random is $1/4$, it follows that the number of male sophomore students is $1/4 \times 160 = 40$. Therefore the number of female sophomore students is $60 - 40 = 20$. The chance of picking a sophomore student at random is $60/160 = 3/8$. The conditional chance of picking a sophomore student given they are female is simply the proportion of female sophomore students out of the total number of female students; this is $20/80$ or $1/4$.
- (c) The conditional chance of picking a male student given that they are sophomore is simply the proportion of male sophomore students out of the total number of sophomore students; this is $40/60 = 2/3$.

Problem 2. In each of the following cases, find the more likely event (that is the event that has higher chance).

- (a) Getting a total of 10 spots on two rolls of a die or getting two cards of the same suit on a couple of draws from the top of a well-shuffled deck.

Solution: With two rolls of a die there are 36 possible pairs of outcomes (like (1,1), (1,2), etc.) and each is equally likely. The number of pairs for which the total number of spots is 10 are (4,6), (6,4) and (5,5). Hence the chance of getting a total of 10 spots is $3/36 = 1/12$.

Getting two cards of the same suit from the top of a well shuffled deck is made up of four mutually exclusive events; namely getting two spades or getting two hearts or getting two diamonds or getting two clubs. The chance of getting two spades is Chance of spades in first draw \times chance of spades in second draw given spades in first draw $= 13/52 \times 12/51 = 3/51 = 1/17$. This is also the chance of getting a pair of hearts or a pair of diamonds or a pair of clubs. Hence the required chance is $4 \times 1/17 = 4/17$ and this is higher than $1/12$, which is the chance of the first event.

- (b) Getting two tails in 4 tosses of a fair coin or getting at least one heads in 4 tosses of a fair coin.

Solution: With 4 tosses of a pair coin there are $2^4 = 16$ possible sequences of heads or tails and each is equally likely. Out of these there are 6 sequences for which we have two tails, namely

$(T, T, H, H), (T, H, T, H), (H, H, T, T), (H, T, H, T), (H, T, T, H), (T, H, H, T)$.

Hence the chance of two tails is $6/16 = 3/8$. The chance of atleast one heads is $15/16$ since each sequence except (T, T, T, T) has at least 1 heads. This is larger than $3/8$. So the second event is more likely.

(5+5 = 10 points)

Problem 3. Formulate appropriate box models.

- (a) A die is rolled 80 times. The total number of times that an even number comes up is like the sum of (i) draws from the box (ii) .
- (b) A quiz has 25 multiple choice questions. Each question has 5 possible answers, one of which is correct. A correct answer is worth 4 points but a point is taken off for each incorrect answer. A student answers all the questions by guessing at random. The total score on the quiz will be like the sum of (iii) draws from the box (iv) .

Put down appropriate numbers for (i) and (iii) and an appropriate box of tickets for (ii) and (iv). (5+5 = 10 points)

Solution: (i) 80 . (ii) one ticket marked 1 and 1 ticket marked 0. (since in each roll of a die an even number comes up with probability $1/2$ and an odd number with probability $1/2$). This is however not the only solution to (ii).

(iii) 25 . (iv) 4 tickets marked -1 and 1 ticket marked 4. (once again the solution is not unique).