Statistics 220/301 **Banerjee**
Winter 2001

Quiz 2

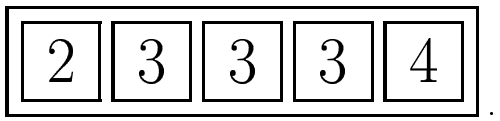
March 22

Your section: _____ Print your name: _____

Sign your name: _____

This is an open book exam. However, you are not allowed to pass any material (such as books, notes, or calculators) to each other. The quiz consists of three problems on pages 2 through 4. Good luck!

Problem 1. Two draws will be made at random *without* replacement from the box



(a) Are the draws independent? Answer with "yes" or "no", and explain.

(b) What is the chance of drawing 2 and then 3?

(c) What is the chance of drawing 2 and then 4?

(d) What is the chance of drawing 2 twice?

Show all your work for parts (b), (c), and (d). (3+2+2+2 = 9 points)

Solution: (a) No, the draws are not independent. If they were, the chance of an outcome on the second draw would not depend on the outcome in the first draw. However, it is easy to see that the chance of a 2 in the second draw given a 2 in the first draw is 0, whereas the chance of a 2 in the second draw given a 4 in the first draw is 1/4.

(b) The chance of drawing a 2 and then a 3 is chance of drawing a 2 times the conditional chance of drawing a 3 in the second draw given a 2 in the first draw. This is simply $(1/5 \times 3/4) = 3/20 = 15\%$.

(c) This is chance of drawing a 2 in the first draw times the conditional chance of drawing a 4 in the second draw given a 2 in the first draw. This is simply $(1/5 \times 1/4) = 1/20 = 5\%$.

(d) The chance of drawing a 2 twice is 0 since we are drawing without replacement.

Problem 2. A die is rolled *three* times.

(a) Find the chance of getting three sixes.

(b) Find the chance of not getting a six.

(c) Find the conditional chance of getting a total of 17 or more spots, given that the first roll was a six and the second was a five.

In each case, explain briefly. (3+3+3 = 9 points)

Solution: (i) With three rolls of a die, the total number of possibilities (of triplets, like (1,3,4), (4,5,6) etc.) is $6 \times 6 \times 6 = 216$, and each is equally likely and therefore has chance 1/216. The only way to get three sixes is to have (6,6,6) and this therefore has chance 1/216.

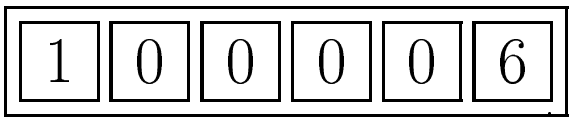
(ii) To not get a 6, it must be the case that we do not get a 6 on the first or the second or the third roll. Thus there are 5 options for the first roll (1,2,3,4,5), 5 for the second and 5 for the third. Thus the number of triplets for which we get no 6's is $5 \times 5 \times 5 = 125$ and thus the required chance is 125/216.

(iii) Given that the first roll was a 6 and the second was a 5, the only way in which the sum of 3 rolls can be atleast 17 is for the third roll to end up in a 6. The chance of getting a 6 in the third roll is 1/6 (the third roll is independent of the first or the second roll). Hence this is the required chance.

Problem 3. You can choose among two options:

(i) You roll a die 50 times; on each roll, if it lands a one or a six you win \$1, if it lands another number nothing happens.

(ii) You draw 50 times at random with replacement from the box



On each draw, you are paid (in dollars) the number on the ticket.

Which option is better for you? Or are they the same? Explain your answer. (7 points)

Solution: The second option is better. With the first option the chance of winning a dollar is exactly 1/3; with the second option you win at least 1 dollar with probability 1/3 and you can actually win 5 more dollars with probability 1/6 (if you draw 6 from the box). Looking at this from the angle of expected values, the average amount that you gain from each roll in the die-rolling experiment is a third of a dollar and so the expected gain from 50 draws is 16 and two-thirds of a dollar. When you draw from the box, the average amount that you gain from each draw is 1 and 1/6'th of a dollar, showing that the expected gain from 50 draws is over 58 dollars.