

## Quiz 3

March 6

Your section: \_\_\_\_\_ Print your name: \_\_\_\_\_

Sign your name: \_\_\_\_\_

This is an open book exam. However, you are not allowed to pass any material (such as books, notes, or calculators) to each other. The quiz consists of four problems each carrying 8 points but the maximum you can score is 25. You have 40 minutes. Good luck!

**Problem 1.** A fair coin is thrown 1000 times.

- (i) Find the chance that the number of heads is between 490 and 520.
- (ii) Find the chance that the percentage of tails is between 48% and 51%.

(4+4 = 8 points)

**Solution:**

- (i) The expected value for the number of heads is 500 (recall that the relevant box model is a 0 - 1 box with 1's corresponding to heads and 1000 draws are made) and the standard error for the number of heads is

$$S.E = \sqrt{1000} * 0.5 = 15.8$$

approximately (the SD of the box is 0.5). Since the number of draws is large the probability histogram for the sum of draws will follow the normal curve. To get the chance that the number of heads is between 490 and 520, convert 490 and 520 to standard units using the expected value and standard error.

$$\frac{490 - 500}{15.8} = -.63 \text{ (approx)}$$

$$\frac{520 - 500}{15.8} = 1.27 \text{ (approx)}$$

To find the chance we then just need to work out the area under the normal curve between -.63 and 1.27. This is left as an exercise.

- (ii) Note that the event that the percentage of tails is between 48 % and 51 % is the same as the event that the number of heads is between 49 % and 52 %. Since there are 1000 tosses, this is the same as saying that the percentage of heads is between 490 and 520, which is what we found in (i). So the chance in (i) is the answer.

**Problem 2.** Six hundred draws will be made at random with replacement from the box  $[-1, 0, 1]$ . The sum of the numbers in the draws that are less than 1 will be around (a), give or take (b) or so. Find appropriate numbers for (a) and (b). Hence compute the chance that the sum of the numbers in the draws that are less than 1 is between -220 and -180.

(8 points)

**Solution:** We only add up the numbers from the 600 draws that are less than 1. This means that whenever we get a 1, we actually count it as a 0 for the purpose of forming a sum. Thus we can think of drawing 600 times at random with replacement from the box  $[-1, 0, 0]$ . The expected value for the sum of 600 draws is  $600 \times \text{average of box} = 600 \times \frac{-1}{3} = -200$ . The standard error (S.E.) is  $\sqrt{600} \times \text{SD of the box } [-1, 0, 0]$ . This is  $\sqrt{600} \times 0.47 = 11.5$  (appx). Hence (a) equals 600 and (b) equals 11.5. To find the chance that the sum of the numbers in the draws is between -220 and -180, we note that the number of draws is large (600) and so we can use the normal approximation. Using the expected value and standard error, convert -220 and -180 to standard units. They are -1.74 and 1.74 respectively. So we need to find the area under the normal curve between -1.74 and 1.74 which is about 92 %.

**Problem 3.** A survey research company has a contract to estimate the percentage of people listening to Spanish-language radio programs in several Southwestern cities. They are using a simple random sample of size 1,000 in Austin, Texas, which has about 500,000 residents. And they are satisfied with the accuracy of the estimates for Austin.

Dallas has about twice the population of Austin, but similar demographics. Decide whether the following statement is true or false, and explain: *to get about the same accuracy in Dallas as in Austin, the company should use a simple random sample of size 2,000.*

( 8 points)

**Solution:** The statement is false. 1000 is a very small fraction of 500,000 and an even smaller fraction of 1,000,000 which is twice of 500,000. So the SE for the sample percentage (which measures the accuracy) is determined essentially by the absolute size of the sample and not the size relative to the population. (Section 4 of Chapter 20 gives a more elaborate explanation) So, to get the same accuracy in Dallas as in Austin we can use a sample of the same size, 1000.

**Problem 4.** Ten thousand draws are made at random from a box with 99 tickets marked 0 and 1 ticket marked 1. True or false and explain:

- (i) The sum will be around 100, give or take 10 or so.
- (ii) There is about a 68 % chance that the sum will be in the range 90 to 110.

**Solution:**

- (i) This is true. The expected value for the sum is

$$10000(\text{number of draws}) \times (1/100)(\text{average of box}) = 100.$$

The SE for the sum is

$$\begin{aligned} \sqrt{10000} \times 1 \times \sqrt{99/100 \times 1/100} \\ = \sqrt{99} = 10, \end{aligned}$$

nearly.

- (ii) This is true as well. This follows directly on noting that 90 in standard units is -1 (appx) and 110 in standard units is 1 (appx) and using the normal curve to estimate the chance.