

Quiz 2

May 17

Your section: _____ Print your name: _____

Sign your name: _____

This is an open book exam. The total number of points is 31, but the maximum you can score is 25. You are not allowed to pass any material (such as books, notes, or calculators) to each other. The quiz consists of TWO problems and you have 35 minutes. Show as much work as possible. Good luck!

Problem 1. Russ tosses a die three times. The die may be supposed to be fair, so that on a single toss each face is as likely to turn up as any other.

- (a) What is the chance that the sum of the spots obtained on these three throws is less than 5 ?
- (b) What is the chance that the sum of the spots obtained on the second and third throws is 11 ?
- (c) What is the chance that the number of spots obtained on the first throw equals the number of spots obtained on the third ?
- (d) What is the conditional chance that Russ got 5 on the first throw given that the sum of the spots obtained on the three throws is 18 ?

(4 + 4 + 4 + 3 = 15 points)

Solutions:

1.(a) There are 216 outcomes all equally likely. The outcomes favorable to getting a total less than 5 are $(1, 1, 1), (1, 2, 1), (2, 1, 1), (1, 1, 2)$. Hence the chance $= 4/216 = 1/54$.

1.(b) Just think about outcomes on second and third throws now. There are 36 different outcomes. The ones favorable to getting a total of 11 spots are $(5, 6), (6, 5)$. Hence the required chance is $2/36 = 1/18$.

1.(c) Now think about the first and third throws only. There are 36 possibilities. The favorable outcomes are $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)$; that is 6 favorable outcomes. So the required chance is $6/36 = 1/6$.

1.(d) Given that the sum of the spots on three throws is 18, it follows that each throw must have resulted in a 6; so the conditional chance of getting a 5 on the first throw is 0.

Problem 2. Susan shuffles a deck of cards thoroughly and places it on the table. She then draws three cards from the top of the pile.

- (a) What is the chance that the first card is an ace, the second card is a king and the third card is a queen ?

Soln: Chance : $(4/52) \times (4/51) \times (4/50)$.

- (b) What is the chance that at least one of the cards drawn is a queen ? **Soln:**
(b) $1 - P(\text{none of the cards is queen})$
 $= 1 - (48/52) \times (47/51) \times (46/50)$.

- (c) What is the chance that the first two cards are hearts ?

Soln: Chance: $(13/52) \times (12/51)$.

- (d) What is the chance that one card is an ace, one a king and one a queen ?

Soln: A bit more challenging than the rest. The event : one ace (A), one king (K), one queen (Q) decomposes disjointly as (AKQ), (AQK), (KAQ), (KQA), (QAK), (QKA), these being the different orders in which you get the cards. From (a) each of these events has probability $64/(52 \times 51 \times 50)$. By the addition rule the required probability is: $6 \times 64/(52 \times 51 \times 50)$.

(4 + 4 + 4 + 4 = 16 points)