Correction to: A Glivenko-Cantelli Theorem and Strong Laws of Large Numbers for Functions of Order Statistics

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CORRECTION TO
A GLIVENKO-CANTELLI THEOREM AND STRONG
LAWS OF LARGE NUMBERS FOR FUNCTIONS
OF ORDER STATISTICS

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Professors Peter Gaenssler and Winfried Stute have kindly brought to my
attention an error in the proof of (B) of Theorem 1 of the above-mentioned paper
(Ann. Statist. 5 (1977) 473–480). The proof given there is valid under the addi-
tional assumption that \( h \) is concave or convex; or if \( h \geq a l \) for some \( a > 0 \). It
is easily seen however that there exist nonnegative, nondecreasing continuous
functions \( h \) which satisfy
\[
\lim \inf_{t \to 0} h(t)/t = 0 \quad \text{and} \quad \lim \sup_{t \to 0} h(t)/t = +\infty .
\]
These functions are not concave or convex or bounded below by any line through
the origin; hence the argument given in the first seven lines of the proof of
Theorem 1 is invalid.

Fortunately, part (B) of Theorem 1 is true as stated (without an additional
convexity or concavity or boundedness assumption as discussed above). Further-
more, (C) if \( \int_0^\infty (1/h) \, dl = \infty \) then
\[
\lim \sup_{n \to \infty} \rho_n(\Gamma_n, I) = +\infty \quad \text{w.p. 1} .
\]
The following simple proof of both (B) of Theorem 1 and (C) is due to Gaenssler
and Stute.

Without loss suppose \( \int_0^\infty (1/h) \, dl = \infty \). Then for any positive integer \( r \) and
\( n \geq N = N(r, \varepsilon) \) the sequence
\[
c_n \equiv \sup \{ t \leq \varepsilon : h(t)^{-1} = 2nr \}
\]
is well defined and
\[
\sum_{n=0}^\infty c_n = (2r)^{-1} \sum_{n=0}^\infty c_n(2(n + 1)r - 2nr)
\geq (2r)^{-1} \int_0^\infty (1/h) \, dl - \text{constant} = \infty .
\]
Thus Borel–Cantelli implies that \( P(\xi_n \leq c_n \text{ i.o.}) = 1; \) and consequently \( P(\xi_n \leq c_n \text{ i.o.}) = 1 \) also. Since \( 1/h \) is continuous and nonincreasing on \((0, \varepsilon)\), this implies that
\[
\rho_h(\Gamma_n, 0) = \sup_{0 < t < 1} (\Gamma_n(t)/h(t)) \geq (nh(\xi_n))^{-1} \geq (nh(c_n))^{-1} = 2r
\]
infinity often w.p. 1, which yields (B) of Theorem 1 as claimed. Similarly,
\[
\rho_h(\Gamma_n, I) \geq (2nh(\xi_n))^{-1} \geq (2nh(c_n))^{-1} = r
\]
infinity often w.p. 1, which proves (C).

It should be noted that (C) together with (A) of Theorem 1 of the paper imply
that finiteness of \( \int_0^\infty (1/h) \, dl \) is both necessary and sufficient for the weighted
Glivenko–Cantelli theorem for \( \Gamma_n \).