1. Centering (written): Let \( \hat{\beta}_0, \hat{\beta}_1 \) be the OLS estimates for the regression of a data vector \( \mathbf{y} \) on a data vector \( \mathbf{x} \), and let \( \hat{\alpha}_0, \hat{\alpha}_1 \) be the OLS estimates for the regression of \( \mathbf{y} \) on the vector \( \tilde{\mathbf{x}} \), where \( \tilde{x}_i = x_i - \bar{x} \).

   (a) Derive expressions for \( \hat{\alpha}_0 \) and \( \hat{\alpha}_1 \) in terms of \( \hat{\beta}_0, \hat{\beta}_1 \) and \( \bar{x} \).

   (b) Before the experiment is run, the data are random. Under our assumptions \( A1 \) and \( A2 \), compare the pre-experimental covariance of \( (\hat{\beta}_0, \hat{\beta}_1) \) to that of \( (\hat{\alpha}_0, \hat{\alpha}_1) \).

2. Practice with matrices (written): Consider an OLS fit of \( \mathbf{y} \) to \( \mathbf{X} \):

   (a) Show that the vector of residuals is given by \( \hat{\mathbf{e}} = (\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T)\mathbf{y} \).

   (b) Show that \( (\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T) \) is idemopotent.

   (c) Show that \( \text{RSS}(\hat{\beta}) = \mathbf{y}^T(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X})\mathbf{y} \).

3. UN11 (Rmd): Load the \texttt{UN11} data from the \texttt{alr4} package and do problem W3.2 from the book, but add the following to 3.2.2: Fit a regression for \texttt{fertility} \( \sim \) \texttt{ppgd}\texttt{p}. Compare the \( R^2 \) values across all three models. Also, for all three models, make a fitted versus residual plot and a qqnorm plot, and comment on whether or not the assumptions of the normal linear model seem to be met in each case. Comment on the use of \texttt{ppgd}\texttt{p} versus \texttt{log(ppgdp)} as a predictor for \texttt{fertility}. 

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