1. Find and sketch the influence curve $IC(x) = d_i T(F; \varepsilon_i - F)$ as a function of $x$ for the following functionals:
   (a) $T(F) = \int x dF(x)$.
   (b) $T(F) = F^{-1}(p), \; 0 < p < 1$, assuming that $F$ is differentiable at $F^{-1}(p)$.
   (c) $T(F) = \frac{1}{1 - 2\alpha} \int_0^{F^{-1}(p)} dp$.

2. Let $F_n^{(-j)}$ be the edf of $X_1, \ldots, X_{j-1}, X_{j+1}, \ldots, X_n$. Define a **pseudo-value** for a statistical functional $T(F)$ by
   \[ T_j = n T(F_n) - (n-1) T(F_n^{(-j)}). \]
   An estimate of $\text{Var}_{F}(T(F_n))$ is \( \hat{\sigma}_n^2 = n^{-2} \sum_{i=1}^{n} (T_j - \bar{T})^2 \). This is called the **jackknife** estimate of variance.
   (a) Suppose that $T(F_n) = n^{-1} \sum_{j=1}^{n} \psi(X_j)$. Show that $n \hat{\sigma}_n^2 \to \text{Var}_F\psi(X)$ in probability.
   (b) Let $T(F) = \frac{1}{2} \left( \inf \{ x : F(x \geq \frac{1}{2} \} + \sup \{ x : F(x) \leq \frac{1}{2} \} \right)$ be the midpoint of the interval of medians of $F$. Find the jackknife estimator of variance and show that when $n$ is even and $F$ is $U(0,1)$, then $n \hat{\sigma}_n^2 \to Y^2 / 4$ in distribution, where $Y \sim \text{Exp}(1)$ (rather than to the correct value $\frac{1}{4}$).
   **Hint:** You may use (without proof) the fact that $n(X_{(k)} - X_{(k-1)}) \overset{d}{\to} Y, \; k = 2, \ldots, n$.

3. Let $f(x; \theta) = f_0(x - \theta)$, where $f_0$ is a differentiable density.
   (a) Show that the likelihood equation is $\sum f_0'(x_i - \theta) / f_0(x_i - \theta) = 0$.
   (b) Determine the Fisher information, and show that it is independent of $\theta$.
   (c) Apply the results in (a) and (b) to
   \[ f_0(x) = \begin{cases} \frac{1 - \varepsilon}{\sqrt{2\pi}} e^{-x^2/2}, & |x| \leq c \\ \frac{1 - \varepsilon}{\sqrt{2\pi}} e^{-|x-c|^2/2}, & |x| > c \end{cases} \]
   where $c$ depends on $\varepsilon$.

4. Let $x_1, \ldots, x_n, y_1, \ldots, y_n$ be samples from $F(x)$ and $F(x - \Delta)$, respectively. In order to test the hypothesis $\Delta = 0$ against $\Delta > 0$, one can use the test statistic
   \[ S_n = (1/n) \sum r_i - (n + 1)/2 \]
   where $r_i$ is the rank of $x_i$ in the combined sample (this is the Wilcoxon test).
   (a) Show that if $\Delta = 0$ we have that