1. Let $X_1, \ldots, X_n$ be iid with density $f(x; \theta)$ where $X_i \in \{1, \ldots, L\}$. Estimate $\theta$ by maximum likelihood, and let $\hat{f} = \left( f(1, \hat{\theta}), \ldots, f(L, \hat{\theta}) \right)^T$. Draw $X_i^*$ from $\hat{f}$, $i = 1, \ldots, n$ and let $p_i^* = \# \{ j : X_j^* = i \} / n$. Finally write $p^* = (p_1^*, \ldots, p_L^*)^T$.

(a) Show that

$$\hat{\theta}(p^*) = \hat{\theta} + \eta(\hat{\theta}) (p^* - \hat{f}) / I_1(\hat{\theta})$$

where $\eta(\hat{\theta}) = \frac{\partial}{\partial \theta} \log \hat{f}$ and $I_1$ is the Fisher information in a single observation.

(b) Deduce that $\text{Var}(\hat{\theta}(p^*)) = (nI_1(\hat{\theta}))^{-1}$.

2. Let $X_1, \ldots, X_n$ be iid with density

$$f(x; \theta) = \left( \frac{x}{\theta} \right)^{\theta A'(\theta)} e^{A(\theta) + B(x)}, x > 0, \theta > 0$$

where $A$ is a twice differentiable function.

(a) Show that the mle $\hat{\theta}$ is the geometric mean of the data.

(b) Consider a trimmed geometric mean, where the smallest and largest 100$\alpha$ % of the sample are removed, and the geometric mean of the remaining values is computed. Find the influence curve of this estimator, sketch it, and show that the estimator is B-robust.

3. Due to the influence of an eccentric (but well-placed) uncle, you have become a rising star at Throgmorton Enterprises. Indeed, Mrs. Throgmorton herself has just given you an important assignment. The company is about to buy some components whose lifetimes are modelled as iid exponential random variables with mean $1/\lambda$. Your task is to estimate $\lambda$ using the manufacturers experimental records. When you arrive at the factory, however, you find that the records only show whether a part lasted longer than 200 hrs. In addition, the general untidiness of the plant makes you suspicious that errors may have crept into the data.

(a) Find the mle $\hat{\lambda}$ based on the manufacturers records.

(b) Find the approximate distribution of this estimator for large $n$.

(c) Determine the gross error sensitivity of $\hat{\lambda}$. 