Problem set 4

1. Let $X$ be a continuous random variable with strictly increasing cdf $F(x)$. Let $Y = F(X)$ (so if $X = 2$ then $Y = F(2)$). What is the cdf of $Y$?

2. Let $S_n = X_1 + \ldots + X_n$ where the $X_i$ are iid positive random variables with finite mean $\mu$. Show that $E(S_m/S_n) = m/n$ for $m < n$.

3. Let $f(x)$ be an integrable function with $0 \leq f(x) \leq 1$ for $0 \leq x \leq 1$. Let $Q$ be a point picked at random in the unit square, and call $Q$ a hit if $Q$ lies below $f(x)$.
   (a) What is the probability of a hit?
   (b) How would you use the result in (a) together with the law of large numbers to compute $\int_0^1 f(x) \, dx$?

4. Consider a box with $r$ red and $b$ black tickets. A ticket is drawn at random, its color noted, and replaced together with one more of the same color.
   (a) What is the probability of drawing black on the second draw? Third? $n^{th}$?
   (b) What is the expected number of black tickets drawn in $n$ trials?
   *Hint:* Let $X_i$ be 1 if the $i^{th}$ ball drawn is black, 0 otherwise.
   (c) Show that the correlation between $X_i$ and $X_j$ is $1/(r+b+1)$ for $i \neq j$.
   (d) Find the variance of the number of black tickets drawn in $n$ trials.