1. Let $F$ be a cdf on $[0,1]$, and define for $\alpha > 1$ 
\[ T(F) = \sum_{x \in [0,1]} (F(x) - F(x^-))^\alpha. \]
(a) Compute the Gateux derivative of $T$ at the uniform distribution $U$ on $[0,1]$.
(b) Show that $\sqrt{n} R_{1,n} = \sqrt{n} (T(F_n) - T(U) - d(T(U; F_n - U))) \not\to 0$ in probability when $\alpha < 3/2$.

2. (a) If $T$ has a linear derivative, show that the kernel $h$ is only defined up to an additive constant, and deduce that we can make $\mu(T, F) = 0$ by choosing $h$ appropriately.
(b) A derivative is called $k$-linear if it can be written 
\[ d T(F; G - F) = \int h(x_1, \ldots, x_k) \prod_{i=1}^k d(G(x_i) - F(x_i)) \]
Show that we can find a kernel $\tilde{h}$ (depending on $F$) such that 
\[ d T(F; G - F) = \int \cdots \int \tilde{h}(x_1, \ldots, x_k) dG(x_1) \cdots dG(x_k) \]
(c) Show that $\int \tilde{h}(x_1, \ldots, x_k) dF(x_i) = 0$.

3. Let $T(F) = \mu_F^2 = (E_F X)^2$. Using the asymptotic theory for statistical functionals, find the limiting distribution of $T(F_n)$ when $\mu_F = 0$.

4. Let $X_1, \ldots, X_n$ be nonnegative random variables with cdf $F(x) = \int_0^x t dG(t)$ where $G$ is a cdf on $[0, \infty)$ with mean $\mu$.
(a) Show that $E\left(\frac{1}{X}\right) = \frac{1}{\mu}$.
(b) Show that $G(x) = \int_0^x \frac{\mu}{y} dF(y)$.
(c) Use the results from (a) and (b) to develop an estimator of $G$, and determine its asymptotic distribution.