

Bayesian kriging

Instead of estimating the parameters, we put a prior distribution on them, and update the distribution using the data.

$$\text{Model: } \mathbf{X}|\theta \sim \mathcal{N}(\beta, \sigma^2 \mathbf{C}(\phi) + \tau^2 \mathbf{I})$$

$$(\mathbf{Z}(s_1) \dots \mathbf{Z}(s_n))^T$$

Matrix with
i,j-element
 $\mathbf{C}(s_i - s_j; \phi)$
(correlation)

measurement
error

$$\theta = (\beta, \sigma^2, \phi, \tau^2)^T$$

Prior and posterior

$$\text{Prior: } f(\theta) = f(\beta)f(\sigma^2)f(\phi)f(\tau^2)$$

$$\text{Posterior: } f(\theta | \mathbf{X} = \mathbf{x}) \propto f(\theta)f(\mathbf{x}|\theta)$$

$$f(\phi | \mathbf{X} = \mathbf{x}) \propto f(\phi) \iint \iint f(\mathbf{x}|\theta)f(\beta)f(\sigma^2)f(\tau^2) d\beta d\sigma^2 d\tau^2$$

Predictive distribution:

$$f(\mathbf{Z}(s_0) | \mathbf{X} = \mathbf{x}) = \int \mathbf{p}(\mathbf{x}; \theta) f(\theta | \mathbf{x}) d\theta$$

kriging predictor

Specifically

Exponential isotropic correlation function:

$$C(h; \phi) = \exp(-\phi h)$$

Default correlation model in geoR.

Prior on β defaults to flat, but can also be normal or fixed

Prior on σ^2 defaults to reciprocal, but can be scaled inverse chisquare or flat

Prior on ϕ can be exponential, uniform, reciprocal, squared reciprocal, or user specified (discrete)

More priors

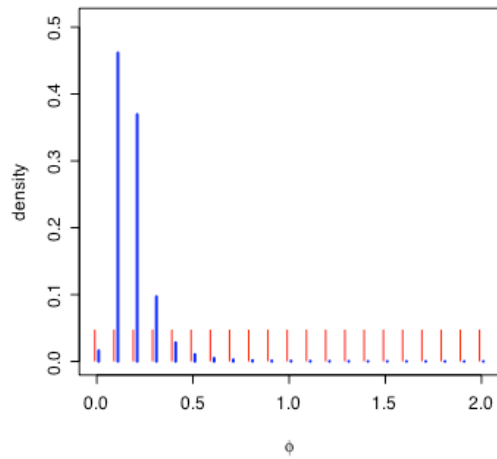
A prior is assigned to τ/σ . Defaults to fixed=0, but can also be uniform or user specified (discrete).

These choices are made for computational reasons. For example, the posterior distribution of σ^2 is inverse chisquared.

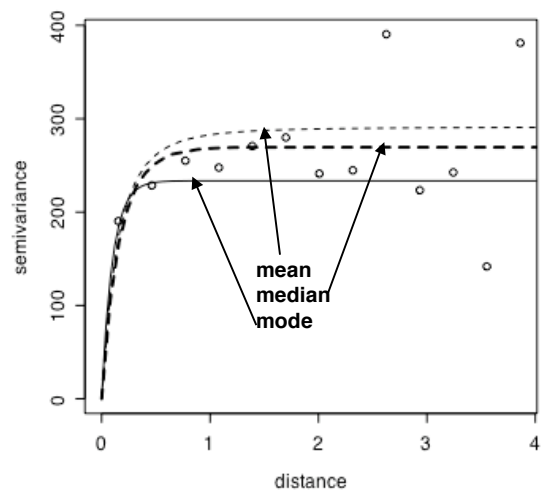
For details, see

<http://www.leg.ufpr.br/geoR/geoRdoc/bayeskrige.pdf>

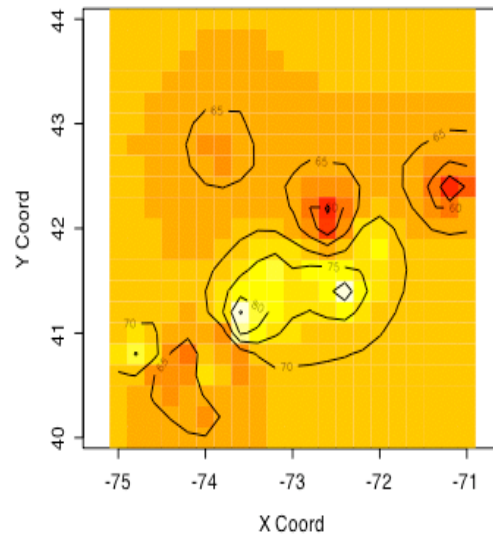
Prior/posterior of ϕ



Variogram estimates

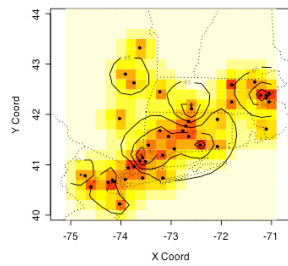


Predictive mean

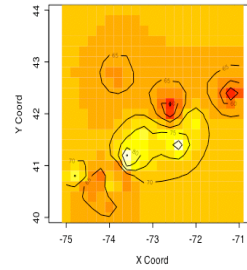


Comparison

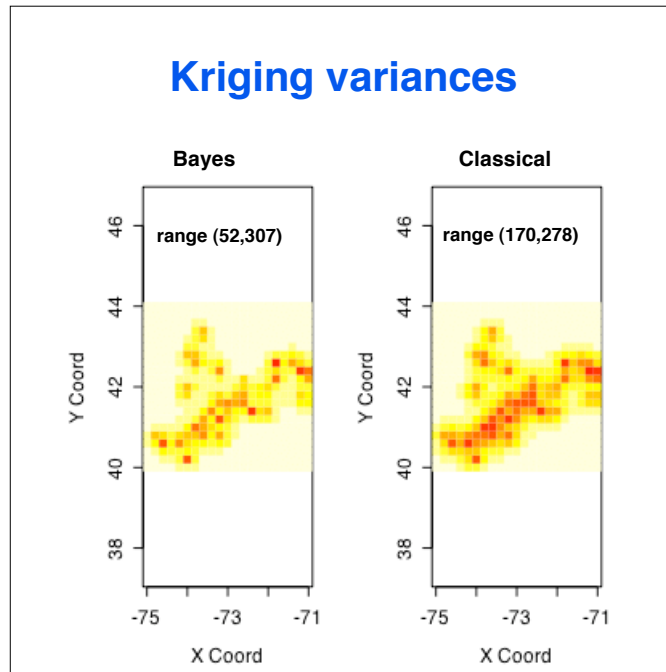
Ordinary kriging



Bayesian kriging



Kriging variances



Introduction to geoR

Commands available at the web site.

“Official” introduction to geoR is

<http://www.leg.ufpr.br/geoR/geoRdoc/vignette/geoRintro.pdf>