Homework problems

(more problems will be added as we go along)

Stat 592, W09

1. For a stationary random field $Z(s); \quad s \in D \subseteq R^2$, observed at sites $s_1,\dots,s_n$, derive the unbiased linear estimator with the smallest variance.

   Hint: Use a Lagrange multiplier to enforce the unbiasedness conditions.

2. For a stationary random field $Z(s); \quad s \in D \subseteq R^2$, observed at sites $s_1,\dots,s_n$, show that the universal kriging estimator for $A(s) = a^T \begin{pmatrix} 1 \\ s \end{pmatrix}$ is unbiased.

3. Compare the variability of simple and ordinary kriging (this can either be done theoretically or by designing an appropriate simulation study).

4. Write R-code to put contours of a kriged surface on a grey-scale background of kriging standard errors.

5. Design a study to compare the plug-in estimate of kriging variance to the real variance of the predictor at a single point. (You do not need to implement the study, just execute a thoughtful design-–see also problem 6).

6. (For those who did problem 5). Implement your study from problem 5.

7. Show that a *d*-dimensional isotropic correlation functions satisfies $\rho(v) \geq -\frac{1}{d}$.

8. Consider the correlation function $\rho(v) = \left(1 - |v| / \phi\right); |v| \leq \phi$. This is a valid correlation function in one dimension. Show that it is not valid in two dimensions.
   Hint: Consider points s_{ij} at a 6x8 grid of size $\phi / \sqrt{2}$. Look at $\text{Var}\sum a_{ij} Z(s_{ij})$ where $a_{ij} = 1$ if $i+j$ even, -1 otherwise.

9. Compare several spatial covariance models graphically, by choosing parameters so that the range/effective range, sill and the nugget are the same for all models.

10. Develop an R program that links the variogram cloud points to the geographic map, so that klicking on a point in the cloud scatter highlights the two corresponding sites, and clicking on a site highlights all the scatter points including that site.

11. Compare the ordinary kriging surface of the Parana data to the universal kriging surface resulting from a second-order mean structure. (Note: while comparing two surfaces it often makes sense to look at the difference between them.)

12. Consider a bivariate process $\mathbf{Z}(s) = (Z_1(s), Z_2(s))^T$ with constant mean $\boldsymbol{\mu}$ and stationary cross-covariance

$$\mathbf{C}(s-s') = \begin{pmatrix} C_{11}(s-s') & C_{12}(s-s') \\ C_{21}(s-s') & C_{22}(s-s') \end{pmatrix}$$

Take observations $\mathbf{z}$ at sites $s_1,\ldots,s_n$.

(a) Show that the kriging predictor at a site s_0 has the form

$$E(Z_1(s_0)|\mathbf{z}) = \mu_1 + \gamma\Sigma^{-1}(\mathbf{z}-\boldsymbol{\mu}).$$

(b) Under what circumstances is the prediction at an observed site equal to the observed value at the site?

13. You may do any of the suggested problems in Lab 2 that you did not do in your lab report.

14. Develop a test for stationarity based on the Bayesian analysis of the deformation model.

15. Analyze the Coal Ash data set at http://www.stat.ncsu.edu/people/fuentes/st790m/lab/coalash.dat (cols 2 and 3 are locations, col 4 has data values)

16. Analyze the Ozone data set at http://www.stat.ncsu.edu/people/fuentes/st790m/lab/ozone.dat (cols 1 and 2 are coordinates, col 3 data values).