1. For a stationary random field \( Z(s); \ s \in D \subseteq R^2 \), observed at sites \( s_1, ..., s_n \), derive the unbiased linear estimator with the smallest variance.

   \textit{Hint:} Use a Lagrange multiplier to enforce the unbiasedness conditions.

2. For a stationary random field \( Z(s); \ s \in D \subseteq R^2 \), observed at sites \( s_1, ..., s_n \), show that the universal kriging estimator for \( A(s) = a^T \left( \frac{1}{s} \right) \) is unbiased.

3. Compare the variability of simple and ordinary kriging (this can either be done theoretically or by designing an appropriate simulation study).

4. Write R-code to put contours of a kriged surface on a grey-scale background of kriging standard errors.

5. Design a study to compare the plug-in estimate of kriging variance to the real variance of the predictor at a single point. (You do not need to implement the study, just execute a thoughtful design—see also problem 6).

6. (For those who did problem 5). Implement your study from problem 5.

7. Show that a \( d \)-dimensional isotropic correlation functions satisfies \( \rho(v) \geq -\frac{1}{d} \).

8. Consider the correlation function \( \rho(v) = \left( 1 - \frac{|v|}{\phi} \right); |v| \leq \phi \). This is a valid correlation function in one dimension. Show that it is not valid in two dimensions.

   \textit{Hint:} Consider points \( s_{ij} \) at a \( 6 \times 8 \) grid of size \( \phi / \sqrt{2} \). Look at \( \text{Var} \sum a_{ij} Z(s_{ij}) \) where \( a_{ij} = 1 \) if \( i+j \) even, \(-1\) otherwise.

9. Compare several spatial covariance models graphically, by choosing parameters so that the range/effective range, sill and the nugget are the same for all models.

10. Develop an R program that links the variogram cloud points to the geographic map, so that clicking on a point in the cloud scatter highlights the two corresponding sites, and clicking on a site highlights all the scatter points including that site.

11. Compare the ordinary kriging surface of the Parana data to the universal kriging surface resulting from a second-order mean structure. (Note: while comparing two surfaces it often makes sense to look at the difference between them.)
12. Consider a bivariate process \( \mathbf{Z}(s) = (Z_1(s), Z_2(s))^T \) with constant mean \( \mu \) and stationary cross-covariance

\[
\mathbf{C}(s - s') = \begin{pmatrix}
C_{11}(s - s') & C_{12}(s - s') \\
C_{21}(s - s') & C_{22}(s - s')
\end{pmatrix}
\]

Take observations \( \mathbf{z} \) at sites \( s_1, \ldots, s_n \).

(a) Show that the kriging predictor at a site \( s_0 \) has the form

\[
E(Z_1(s_0) | \mathbf{z}) = \mu_1 + \Sigma^{-1}(\mathbf{z} - \mu).
\]

(b) Under what circumstances is the prediction at an observed site equal to the observed value at the site?

13. You may do any of the suggested problems in Lab 2 that you did not do in your lab report.

14. Develop a test for stationarity based on the Bayesian analysis of the deformation model.

15. Analyze the Coal Ash data set at
http://www.stat.ncsu.edu/people/fuentes/st790m/lab/coalash.dat (cols 2 and 3 are locations, col 4 has data values)

16. Analyze the Ozone data set at
http://www.stat.ncsu.edu/people/fuentes/st790m/lab/ozone.dat (cols 1 and 2 are coordinates, col 3 data values).