

Homework Assignment 1

CSSS 536: Logistic Regression and Log-Linear Modeling for the Social Sciences

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Autumn Quarter 2002

Due before class on October 18

Problem 1

The fraction of males in a given population is 0.49. What is the probability of sampling 22 males in a random sample of size 30? How about sampling 16 males in a random sample of size 30?

Problem 2

Consider two random variables X and Y . X can take values equal to 1, 2, or 3, and Y can take values equal to 1, 2, 3, 4. We know the following joint probabilities:

- $f_{X,Y}(1, 1) = 0.05$
- $f_{X,Y}(1, 2) = 0.08$
- $f_{X,Y}(1, 3) = 0.13$
- $f_{X,Y}(1, 4) = 0.03$
- $f_{X,Y}(2, 1) = 0.07$
- $f_{X,Y}(2, 3) = 0.16$
- $f_{X,Y}(2, 4) = 0.04$
- $f_{X,Y}(3, 1) = 0.06$
- $f_{X,Y}(3, 2) = 0.07$
- $f_{X,Y}(3, 3) = 0.15$
- $f_{X,Y}(3, 4) = 0.06$

Using what you know about the relationships between marginal, conditional, and joint probability functions and the information above, compute the following quantities:

1. $f_X(1)$
2. $f_X(2)$
3. $f_X(3)$
4. $f_Y(1)$

5. $f_Y(2)$
6. $f_Y(3)$
7. $f_Y(4)$
8. $f_{X,Y}(2,2)$
9. $f_{X|Y}(1|2)$
10. $f_{X|Y}(3|4)$
11. $f_{Y|X}(4|3)$
12. $f_{Y|X}(2|2)$

Hint: It may help to arrange the available joint probabilities in a table.

Problem 3

Consider the following dataset: $\mathbf{y} = (1, 0, 0, 1, 0, 0, 0, 0)$ that is a series of independent realizations of a Bernoulli distributed random variable. Plot the likelihood function for the Bernoulli parameter π given \mathbf{y} . If the experiment was repeated, roughly what fraction of the observations would you expect to be successes ($y = 1$)? Why?

Problem 4

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 5 & 0 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 1 & 2 & -3 \\ 7 & 1 & -1 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \mathbf{D} = \begin{bmatrix} 9 & 10 & 1 \\ 1 & 3 & 7 \end{bmatrix} \mathbf{e} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{f} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Compute each of the following matrices if it is defined. If it is not defined state why it is not defined.

1. $\mathbf{A} + \mathbf{B}$
2. \mathbf{AB}
3. $\mathbf{B} + \mathbf{D}$
4. \mathbf{AB}'
5. $\mathbf{D}'\mathbf{f} + \mathbf{f}$
6. \mathbf{De}
7. \mathbf{CC}
8. \mathbf{CA}
9. \mathbf{AC}
10. $(\mathbf{C} + \mathbf{A})\mathbf{e}$
11. $\mathbf{e}'\mathbf{e}$
12. \mathbf{ee}'
13. $\mathbf{f}'\mathbf{f}$
14. \mathbf{ff}'
15. \mathbf{ACA}
16. $\mathbf{e}'\mathbf{Ce}$