CORRECTIONS AND CHANGES FOR:
EMPIRICAL PROCESSES
WITH APPLICATIONS TO STATISTICS (Wiley, 1986)

by

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March 3, 1989
1. Introduction

Since publication of our book

Empirical Processes with Applications to Statistics

in 1986, we have become aware of several mathematical errors and a number of typographical and other minor errors. Our purpose here is to give corrections of the errors of which we are currently aware. Although we would now do many things differently, we have not made any of that type of revisions here.

We encourage readers finding further errors to let us know of them.

We owe thanks to the following friends, colleagues, reviewers, and users of the book for telling us about errors, difficulties, and shortcomings: N. H. Bingham, M. Csörgő, S. Csörgő, Kjell Doksum, Peter Gaenssler, Richard Gill, Paul Janssen, Keith Knight, D. W. Müller, David Pollard, Peter Sasieni, and Ben Winter.

We owe special thanks to Peter Gaenssler for providing us with a long list of typographical errors which provided the starting point for section 3 here.

The corrections of chapters 7 and 23 given in section 2 were aided by discussions and correspondence with Richard Gill and Ben Winter (in the case of chapter 7) and Keith Knight (in the case of section 23.3).

A list of reviews is given in section 6.

June 22, 1989
2. Major changes and revisions

Here we give substantial corrections and revisions of section 7.3 (pages 304 - 306) and section 23.3 (pages 767 - 771).

2.1. Revision and correction of section 7.3.

The last two lines (pp. 305, -7 and -6) of the proof of (1) of theorem 1, page 304 are false. Hence there are also difficulties in the cases (i) - (v) on pages 305-306. The follow revision of section 7.3 should replace that entire section. As indicated in the following text, these results are due to Peterson (1977), Gill (1981), and Wang (1987).

We owe thanks to Richard Gill and Ben Winter for pointing out these difficulties and for correspondence concerning their solution.

Section 7.3, pages 304 - 306, should be replaced by the following:

3. CONSISTENCY OF $\hat{\Lambda}_n$ AND $\hat{F}_n$

In this section we use the representations of Theorem 7.2.1 and continuity of the product integral map $\mathfrak{E}$ which takes $\Lambda$ to $F$ (see section B.6 and especially example B.6.1, page 898) to establish weak and strong consistency of $\hat{\Lambda}_n$ and $\hat{F}_n$. Our first result gives strong consistency of both $\hat{\Lambda}_n$ and $\hat{F}_n$ on any interval $[0, \theta]$ with $0 < \tau \equiv \tau_H \equiv H^{-1}(1)$.

Theorem 1. Suppose that $F$ and $G$ are arbitrary df's on $[0, \infty)$. Recall $\tau \equiv \tau_H \equiv H^{-1}(1)$ where $1 - H \equiv (1 - F)(1 - G)$. Then for any fixed $\theta < \tau$

(1) $\|\hat{F}_n - F\|_{0, \theta}^{\theta} \to_{a.s.} 0$ as $n \to \infty$

and

(2) $\|\hat{\Lambda}_n - \Lambda\|_{0, \theta}^{\theta} \to_{a.s.} 0$ as $n \to \infty$.

The following theorems strengthen (1) of theorem 1 in different directions.

Theorem 2. Suppose $F$ and $G$ are df’s on $[0, \infty)$ with $\tau \equiv \tau_H \equiv H^{-1}(1)$ satisfying either $H(\tau-) < 1$ or $F(\tau-) = 1$. Then

(3) $\sup_{0 \leq t \leq \tau} |\hat{F}_n(t) - F(t)| = \|\hat{F}_n - F\|_{0, \tau}^{\tau} \to_{a.s.} 0$ as $n \to \infty$,

and, with $T \equiv Z_{n:n}$,

(4) $\sup_{0 \leq t \leq T} |\hat{F}_n(t) - F(t)| = \|\hat{F}_n - F\|_{0, T}^{T} \to_{a.s.} 0$ as $n \to \infty$.

The following theorem is more satisfactory since $F$ and $G$ are completely arbitrary; the price is that the consistency is in probability (and the supremum in (5) is just over the interval $[0, \tau]$).

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Theorem 3. (Wang). Suppose that $F$ and $G$ are completely arbitrary. Then

$$\sup_{0 \leq t < \tau} |F_n^\ast(t) - F(t)| \to_p 0 \quad \text{as } n \to \infty,$$

and, with $T \equiv Z_{n:n}$,

$$\sup_{0 \leq t \leq T} |F_n^\ast(t) - F(t)| \to_p 0 \quad \text{as } n \to \infty.$$

Open Question 1. Does Wang’s theorem 3 continue to hold with $\to_p$ replaced by $\to_{a.s.}$? (The hard case not covered by theorem 2 is $F(\tau-) < 1$, $G(\tau-) = 1$.)

Recall that for an arbitrary hazard function $\Lambda$ (of a df $F$ on $R^+$), the (product integral) or exponential map $E(\Lambda)$ recovers $1 - F$:

$$1 - F(t) = E(\Lambda)(t) \equiv \prod_{0 \leq s \leq t} (1 - d\Lambda)$$

$$= \exp(-\Lambda^c(t)) \prod_{0 \leq s \leq t} (1 - \Lambda(s));$$

see section B.6 and example B.6.1. Our proofs of theorems 1 - 3 will use the following basic lemma which is due to Peterson (1977), Gill (1981), and, in the present form, Wang (1987).

Lemma 1. (Continuity of the product integral map $E$). Suppose that $\{g_n\}_{n \geq 0}$ is a sequence of nondecreasing functions on $A = [0, \tau]$ or $[0, \tau)$ satisfying $\Delta g_0 < 1$, and set $h_n = E(-g_n)$, $n = 0, 1, \cdots$. If

$$\sup_{t \in A} |g_n(t) - g_0(t)| \to 0 \quad \text{as } n \to \infty,$$

then

$$\sup_{t \in A} |h_n(t) - h_0(t)| \to 0 \quad \text{as } n \to \infty.$$

Proof of theorem 1. Now $\|H_n - H\| \to_{a.s.} 0$ by Glivenko-Cantelli, so that $\|H_{n-} - H\| \to_{a.s.} 0$ also. Thus for any fixed $t \leq \theta$ we have a.s. that

$$|\hat{\Lambda}_n(t) - \Lambda(t)| \leq \int_{0}^{t} |(1 - H_{n-})^{-1} - (1 - H)^{-1}| \, dH_n^1$$

$$+ |\int_{0}^{t} (1 - H)^{-1} \, d(H_n^1 - H^1)|$$

(a) $\to_{a.s.} 0 + 0 = 0$

by the Glivenko Cantelli theorem and $H(t-) \leq H(\theta) < 1$ for the first term, and by the SLLN for the second term. Since $\hat{\Lambda}_n$ and $\Lambda$ are $\uparrow$, the standard argument of (3.1.83) improves (a) to (2).
1. Proof of theorem 2. First suppose $H(\tau-) < 1$. Then as in (a) of the proof of theorem 1,

\[
|\hat{\Delta}_n(t) - \Lambda(t)| \leq \int_0^t \{(1 - H_{n-})^{-1} - (1 - H^{-1})\} dH_n^1 \]
\[
+ \int_0^t (1 - H^{-1}) d(H_n^1 - H^1)
\]

where the first term converges to zero uniformly on $[0, \tau]$ by the Glivenko - Cantelli theorem since $1 - H(\tau-) > 0$ and $H_n^1(\tau) \leq 1$. Now the second term: for $0 \leq t \leq \tau$,

\[
\int_0^t \frac{1}{1 - H_-} d(H_n^1 - H^1) \]
\[
\leq \frac{H_n^1(t) - H^1(t)}{1 - H(t-)} - \int_0^t (H_n^1(s) - H^1(s)) d\left(\frac{1}{1 - H(s-)}\right) \]
\[
\quad + \frac{\Delta H_n^1(\tau) - \Delta H^1(\tau)}{1 - H(\tau-)} \]
\[
\leq 2 \frac{\|H_n^1 - H^1\|_0^2}{1 - H(\tau-)} + \left| \frac{\Delta H_n^1(\tau) - \Delta H^1(\tau)}{1 - H(\tau-)} \right|
\]

$\to_{a.s.} 0 + 0 = 0$,

so the second term converges to zero a.s. uniformly in $t \in [0, \tau]$. Hence

(a) \[\|\Delta_n - \Lambda\|_0^\infty = \sup_{0 \leq t \leq \tau} |\hat{\Delta}_n(t) - \Lambda(t)| \to_{a.s.} 0.\]

If $\Delta \Lambda(\tau) < 1$, then (3) follows from lemma 1. If $\Delta \Lambda(\tau) = 1$ (so $F(\tau) = 1$), then lemma 1 and (a) imply that

\[
\sup_{0 \leq t \leq \tau} |\hat{F}_n^\circ(t) - F(t)| \to_{a.s.} 0
\]

and

\[
0 \leq 1 - \hat{F}_n^\circ(\tau) \leq 1 - \Delta \hat{\Delta}_n(\tau)
\]

$\to_{a.s.} 1 - \Delta \Lambda(\tau) = 0 = 1 - F(\tau)$,

so again (3) holds.

Now suppose that $F(\tau-) = 1$. Given $\varepsilon > 0$, choose $\theta < \tau$ such that $F(\theta) > 1 - \varepsilon$. For $\theta \leq t \leq \tau$ both

\[
\hat{F}_n^\circ(\theta) \leq \hat{F}_n^\circ(t) \leq 1
\]

and

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Hence

\[ \|\hat{F}_n - F\|_\theta^p \leq \max\{\varepsilon, 1 - \hat{F}_n(\theta)\} \]

(b) \(\rightarrow_{a.s.} \max\{\varepsilon, 1 - F(\theta)\} = \varepsilon\)

by (1). Since \(\varepsilon\) is arbitrary, (1) and (b) imply (3) in this case \(F(\tau-) = 1\) too.

Since \(T = Z_{n:U} \leq \tau\) a.s., (4) follows from (3).

Proof of theorem 3. We first suppose \(\theta \leq \tau\) with \(F(\theta-) < 1\), and show that

(a) \(\sup_{0 \leq t < \theta} |\hat{A}_n(t) - \Lambda(t)| \rightarrow_p 0\) as \(n \rightarrow \infty\)

and

(b) \(\sup_{0 \leq t < \theta} \|\hat{F}_n(t) - F(t)\| \rightarrow_p 0\) as \(n \rightarrow \infty\).

Let \(D_n = \hat{A}_n - \Lambda\). Then, with \(T = Z_{n:U}\), \(D_n^T = \{D_n(t \ T) : t \geq 0\}\) is a square integrable martingale with predictable variation process

(c) \(\langle D_n^T \rangle(t) = \int_0^t \frac{1 - \Delta \Lambda(s)}{n(1 - H_n(s-))} d\Lambda(s)\).

Now

(d) \(\langle D_n^T \rangle(\theta-) \rightarrow_{a.s.} 0\).

To see this, let \(\varepsilon > 0\), and choose \(\sigma < \theta\) so that \(\Lambda(\theta-) - \Lambda(\sigma) < \varepsilon\), and hence \(H(\sigma) < 1\) also. Then

\[ \langle D_n^T \rangle(\theta-) - \langle D_n^T \rangle(\sigma) = \int_{(\sigma, \theta)} 1_{[T \geq s]} \frac{1 - \Delta \Lambda(s)}{n(1 - H_n(s-))} d\Lambda(s) \]

\[ \leq \Lambda(\theta-) - \Lambda(\sigma) < \varepsilon, \]

and, by the Glivenko - Cantelli theorem

\[ n\langle D_n^T \rangle(\sigma) = \int_0^\sigma \frac{1 - \Delta \Lambda(s)}{1 - H_n(s-)} d\Lambda(s) \]

\[ \rightarrow_{a.s.} \int_0^\sigma \frac{1 - \Delta \Lambda(s)}{1 - H(s-)} d\Lambda(s) < \infty. \]

Therefore

\[ \langle D_n^T \rangle(\sigma) \rightarrow_{a.s.} 0 \]

and

\[ \limsup_{n \rightarrow \infty} \langle D_n^T \rangle(\theta-) \leq \varepsilon \quad a.s. \]
Since $\varepsilon > 0$ is arbitrary, (d) holds.

By Lenglart's inequality B.4.1,

\[(e) \quad \sup_{0 \leq t < \theta} |D_n^T(t)| \to_p 0 \quad \text{as } n \to \infty.\]

Since we also have (recall $T = Z_{n:n}$)

\[\{\Lambda(\theta-) - \Lambda(T)\}I_{[T < \theta]} \to_a.s. 0 \quad \text{as } n \to \infty\]

in view of $F(\theta-) < 1$, (a) holds.

Now (a) implies that for every subsequence $\{n'\}$ there is a further subsequence $\{n''\} \subset \{n'\}$ so that

\[(f) \quad \sup_{t < \theta} |\Lambda_n^{n''}(t) - \Lambda(t)| \to_a.s. 0 \quad \text{as } n \to \infty.\]

But by continuity of $E$ given by lemma 1, it follows from (f) that

\[(g) \quad \sup_{0 \leq t < \theta} |\mathcal{F}_n^{n''}(t) - F(t)| \to_a.s. 0,\]

and hence (b) holds when $F(\theta-) < 1$.

To complete the proof of (5), it remains only to consider the case $F(\tau-) = 1$. But then (5) follows from (3).

To prove (6), consider the two cases $H(\tau-) = 1$ and $H(\tau-) < 1$: If $H(\tau-) = 1$, then $T = Z_{n:n} < \tau$ a.s. and hence (6) follows from (5). If $H(\tau-) < 1$, then (6) follows from (4).

Proof of lemma 1. By (7) and $\Delta g_0 < 1$ we can assume that

(a) $\Delta g_n < 1$ for $n = 1, 2, \cdots$.

Since $g_n$ are nondecreasing, finite, and (a) holds, it is easy to verify that $h_n > 0$, $n = 0, 1, \cdots$. For $t \in A$ and $\varepsilon > 0$, define (note (B.5.3))

(b) $g_n^\varepsilon(t) \equiv g_n^\varepsilon(t) - \sum_{s \leq t} \log(1 - \Delta g_n(s)) 1_{[|\Delta g_n(s)| \leq \varepsilon]}$

and

(c) $\bar{g}_n^\varepsilon(t) \equiv - \sum_{s \leq t} \log(1 - \Delta g_n(s)) 1_{[|\Delta g_n(s)| > \varepsilon]}$

so that

(d) $g_n^\varepsilon(t) + \bar{g}_n^\varepsilon(t) = - \log h_n(t)$.

Now $\bar{g}_n^\varepsilon(t)$ is the sum of at most a finite number of terms. Thus by (7) for every $\varepsilon > 0$ with

(e) $\varepsilon \in \{ a < 1/2 : \Delta g_0(t) \neq a \text{ for all } t \in A \}$
it follows that
\[
(f) \quad \sup_{t \in A} \left| \sum_{s \leq t} \Delta g_n(s) \right|_{\|A g_n(s)\| > \epsilon} - \sum_{s \leq t} \Delta g_0(s) \right|_{\|A g_0(s)\| > \epsilon} \to 0
\]
as \( n \to \infty \) and
\[
(g) \quad \sup_{t \in A} \left| g^n(t) - g^0(t) \right| \to 0 \quad \text{as} \quad n \to \infty .
\]
But note that
\[
\left| g^n(t) - g^0(t) \right|
\leq \left| g^n(t) - g^c_n(t) \right| - \sum_{s \leq t} \Delta g_n(s) \right|_{\|A g_n(s)\| \leq \epsilon} \right| + \left| g^c_n(t) + \sum_{s \leq t} \Delta g_n(s) \right|_{\|A g_n(s)\| \leq \epsilon} \right| - g^0(t) - \sum_{s \leq t} \Delta g_0(s) \right|_{\|A g_0(s)\| \leq \epsilon} \right|
+ \left| g^c_n(t) - g^0(t) \right| - \sum_{s \leq t} \Delta g_0(s) \right|_{\|A g_0(s)\| \leq \epsilon} \right|
\leq \left| \sum_{s \leq t} \left\{ \log(1 - \Delta g_n(s)) + \Delta g_n(s) \right|_{\|A g_n(s)\| \leq \epsilon} \right| + \left| \sum_{s \leq t} \Delta g_n(s) \right|_{\|A g_n(s)\| > \epsilon} \right| - \sum_{s \leq t} \Delta g_0(s) \right|_{\|A g_0(s)\| > \epsilon} \right| + \left| g_n(t) - g_0(t) \right|
+ \left| \sum_{s \leq t} \log(1 - \Delta g_0(s)) + \Delta g_0(s) \right|_{\|A g_0(s)\| \leq \epsilon} \right| - \sum_{s \leq t} \Delta g_0(s) \right|_{\|A g_0(s)\| > \epsilon} \right| + \left| g_0(t) \right|
\leq \epsilon (\log g_0(t) + \left| g_0(t) \right|)
+ \left| \sum_{s \leq t} \Delta g_n(s) \right|_{\|A g_n(s)\| > \epsilon} - \sum_{s \leq t} \Delta g_0(s) \right|_{\|A g_0(s)\| > \epsilon} \right|.
\]

Therefore, for every \( \epsilon \) satisfying (e), (f) yields
\[
\limsup_{n \to \infty} \sup_{t \in A} \left| g^n(t) - g^0(t) \right| \leq 2 \epsilon g_0(t)
\]
and hence, by (d) and (g),
\[
(i) \quad \limsup_{n \to \infty} \sup_{t \in A} \left| \log h_n(t) - \log h_0(t) \right| \leq 2 \epsilon g_0(t) .
\]
Since \( \epsilon \) is arbitrary, (i) implies (8). □
2.2 Revision and correction for section 7.7. Weak convergence \( \Rightarrow \) of \( \mathcal{B}_n \) and \( \mathcal{X}_n \) in \( \| \cdot \|_T \)-metrics

On page 325, exercise 3: The displayed equation should read as

\[
(1 - K)/(1 - F) = \left\{ 1 + \int_0^\infty C dF \right\}^{-1}
\]

And then: "Hence \((1 - K)/(1 - F)\) is \( \downarrow \)."

2.3 Revision and correction of section 23.3. The Shorth.

There is an error here in the grouping of the \( n^{1/6} \) factor leading to (i) on page 768; and exercise 1 on page 771) is not correct. The following correction is perhaps the simplest. A different, somewhat longer correction was suggested to us by Keith Knight. (Knight's alternative correction changes the "centering" in the definition of \( M_n \) in (6) from \( 2 F^{-1}(1-a) \) to \( F_n^{-1}(1-a) - F_n^{-1}(a) \).

Begin on page 768 just after (g):

Moreover, since \( g' \) exists and is continuous,

\[
\sup_{|t| \leq K} |g(1-a + \frac{A}{n^{1/3}}) B_n(1-a + \frac{A}{n^{1/3}}) - g(1-a) n^{1/6} B_n(1-a + \frac{A}{n^{1/3}})|
\]

\[
\leq \left\{ \sup_{|t| \leq K} |g(1-a + \frac{A}{n^{1/3}}) - g(1-a)| \right\} \times \left\{ \sup_{|t| \leq K} |B_n(1-a + \frac{A}{n^{1/3}})| \right\}
\]

\[
\leq \left\{ n^{1/4} \sup_{|t| \leq K} |g(1-a + \frac{A}{n^{1/3}}) - g(1-a)| \right\} \times \left\{ n^{-1/12} \sup_{|t| \leq K} |B_n(1-a + \frac{A}{n^{1/3}})| \right\}
\]

(h) \( = o(1) O(1) \) a.s.

(i) \( = o(1) \) a.s.

continue on page 769, line 1.

Correction of Exercise 23.3.1, page 771. Replace the present exercise 1 by the following:

Exercise 1. Show that for any \( 0 \leq K < \infty, 0 \leq A < \infty, \) and \( 0 \leq a < 1 \) we have

\[
n^{-1/12} \sup_{|t| \leq K} |B_n(a + K/n^{1/3})| = O(1) \text{ a.s.}
\]

(Knight's alternative correction for this section involves the following alternative

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**Exercise 1'.** Show that for any $0 \leq K < \infty$, $0 \leq A < \infty$, and $0 \leq a < 1$ we have

$$\sup_{|t| \leq K} n^{1/12} |B_n(a + K/n^{1/3}) - B_n(a)| = O(1) \quad \text{a.s.}$$
3. Typographical and spelling errors, and minor changes

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169 1 \quad 14.1.4 \rightarrow 4.1.1
169 -12 \quad 4.1.2 \rightarrow 4.1.5
169 -1 \quad \vec{P} \rightarrow \vec{P} \; \rightarrow 4.1.2 \rightarrow 4.1.5
195 1 vector \rightarrow matrix; constant \rightarrow constants
224 (32) \quad G_n^2 \rightarrow G^2
224 (32)+1 change G_n \rightarrow_d G \text{ to } G_n^2 \rightarrow_d G^2
224 (33) change P(G > \lambda) \text{ to } P(G^2 > \lambda)
262 (25)+3 \quad d \Lambda(x) = \rightarrow d \Lambda(x) \equiv
264 (6) \quad \mathbb{1}[X_i \leq \gamma] \rightarrow \mathbb{1}[X_i \leq \gamma]
264 (6) \quad 1 \leq i \leq n. \rightarrow 1 \leq i \leq n_j
265 (14) \quad X_i \rightarrow X_{ni} \text{ twice}
266 \quad \text{7 - 8} \quad X_i \rightarrow X_{ni} \text{ throughout}
270 (32) -1 change (A.9.6) to (A.9.16)
272 (40) \quad X_i \rightarrow X_{ni} \text{ twice}
273 (1) \quad X_i \rightarrow X_{ni} \text{ twice}
274 -3 \quad \psi(x) = x^2 \rightarrow \psi(x) = x
275 (9) \quad \| \cdot \|_0^1 , \| \cdot \|_0 \rightarrow \| \cdot \|_1^0 , \| \cdot \|_0
276 (1) \quad X_i \rightarrow X_{ni}
279 (9) \quad N. \rightarrow \mathcal{K} \text{ on RHS}
282 (21) delete garbage before =
294 (4)+3 \quad \tau \equiv \tau_H = \tau_F \max \tau_G \rightarrow \tau \equiv \tau_H = \tau_F \min \tau_G
304-6 see section 2
323 2 change "proof of (10)" to "proof of (9)"
325 Exercise 3 See section 2
425 (15)+5 Mason (1981) \rightarrow Mason (1981b)
425 -1 Mason (1981) \rightarrow Mason (1981b)
478 Exercise 4. +1 Anderson's \rightarrow Anderson's
492 -1 Esseen \rightarrow Ess'éen
545 2 (18) \# \mathcal{U}_n \rightarrow \mathcal{U}_n^\sharp
558 section title \quad \mathcal{K}_n \rightarrow \mathcal{K}
584 (3)-1 \quad (\log_2 n)^{1/4} \sqrt{n} \sqrt{n} \rightarrow (\log_2 n \log n)^2/n^{1/4}
604 (2)+10 \quad n \rightarrow t
604 (2)+11 \quad t \rightarrow n
661 (4)+1 \quad \psi \rightarrow \psi_n \text{ twice}
661 (9)+1 in the next section \rightarrow in section 4
662 (12) \quad = \rightarrow =

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688 (1)-1 since the ... since for the ...
695 (3)+2 \[
\frac{k}{n} \to \frac{k}{n+1}
\]
696 (3)+1 (3.7.4) \to (3.6.4)
697 (15) \[
0 \leq t \leq 1
\]
698 (21) \[
\int_{0}^{p_{n,i+1}} \to \int_{0}^{p_{n,i+1}}
\]
699 (7) \[
\int_{0}^{1} \to \int_{0}^{t}
\]
746 +5 change to: The definition of \( \mathcal{S}_n \) is found first in Smirnov (1947); see also Butler (1969).
747 (11)+2 change to: Smirnov (1947) and Butler (1969) give an expression for the exact distribution.
771 2 \( t \) missing just before \( K \)
778 (16)+4 Wang \to Yang
790 (4) \( \pi \to \Pi \)
802 (d) \( 2.1_{[T_2, \infty]} \to 2.1_{[T_2, \infty]} \)
804 (j)-1 \( F \) should go with \( S_1 \) and \( S_m \) as a subscript
819 -3 \( (e^x - 1 - x^2) \to (e^x - 1 - x) \)
821 -2 nonidentically \to not identically
821 -3 combinations of ... \to combinations of a function of ...
844 (6) \( \sqrt{2s_n} \to \sqrt{2} s_n \)
856 (21)+2 Steinback \to Steinebach
859 -6 Renyi \to Erdős and Rényi
890 (8)+2 replace \( A^c(t) = \sum_{s \leq i} \Delta A(s) \)
by \( A^c(t) = A(t) - \sum_{s \leq i} \Delta A(s) \)
896 (2) \[ dX \to dX^i \]
896 (3) \[ dX \to dX^i \]
897 (6) \[ \int_{[0,r]} \to \int_{[0,t]} \]
898 +3 and +4 \[ (0,r] \to [0,t] \]
903 Bretagnolles enchantillon \to echantillon
904 Burk \to Burke
910 Hu tal \to tail
915 Steinbach Steinbach \to Steinebach
916 Rényi theroy \to theory
925 Wang \to Yang
925 Steinbach \to Steinebach
929 -7, right 877 \to 878
936 9, left Rebelledo \to Rebolledo
938 17, left 676 \to 677

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### 4. Accent mark revisions

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5. Solutions of "Open Questions"

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6. Added References and Review List

6.1. References for Revisions


Martynov, G. V., (1978). *Omega-Square criteria*, Nauka, Moscow. in Russian


6.2. References for Solved Problems


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6.3. List of Reviews


