Name: ________________________________

1. Please write your name in the above space.

2. There are 6 practice questions. On the real exam there will be only 4. **You need to do all 4 questions.** All questions are of equal value (but not necessarily of equal difficulty).

3. Do not turn the page until so instructed. (You will have 90 minutes to work after the examination has been discussed with you.)

4. You may use your crib sheet and the statistical tables provided. Otherwise this is a closed book examination.

5. If you do not have enough room for your work in the place provided, use the back of a nearby page. (However, be sure to mark clearly which problem the material on the back of any page refers to.) If you pull the pages apart, sign all pages.

6. Answers should unambiguously state, in words, the approach taken. You should show your work so that partial credit can be given. Poorly described solutions will be penalized.

7. Good luck!

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Question 1) Power Shortages in Western States  (25 points)

Washington State regulators of hydroelectric power employ a weather prediction strategy to guide their decisions about selling power for the next year to California. They forecast the overall rainfall, which determines the amount of hydroelectric power the state can produce, and adapt their decisions to produce power accordingly.

If the rainfall in the upcoming year is good then the regulator will have a lot of water to produce power, and can sell much of the power to California. If the rainfall is poor, and the regulator has sold a lot of power to California then the reservoirs will be reduced leading to high in-state power costs and environmental problems. In any year the rainfall is either good or bad, and the regulator can either sell a fixed amount of power to California or he can not.

Suppose that a regulator accurately predicts a good year 80% of the time (i.e., predicts that the year will be good in 80% of the years that turn out to be good years). The same regulator accurately predicts a bad year 60% of the time.

In reality, eight of the last ten years have been good years.

You should construct a probability tree for the process on the next page to help in answering questions b) - d).

a) (5 points)

What is the probability that the next year will be good? What concept of probability have you applied to decide this?

b) (5 points)

What is the probability that the regulator predicts a good year and the year is also good?

c) (5 points)

What is the probability that the regulator will say that the next year will be a good year?
d) (5 points)

What is the probability that the regulator predicts the next year accurately?

(5 points) Construct your probability tree below:
Question 2) Incoming Notebooks  (25 points)

The Dean of the College of Arts and Sciences is thinking about requiring next year’s incoming freshman class to buy a notebook computer for school work.

To see if the current students are generally in favor of this idea, he randomly selects 9 of the current freshman class and asks them personally.

Suppose that, in fact, 55% of the current freshman class is in favor of notebooks. The Dean does not know this, although you may use this figure in your computations here.

a) (3 points)

What is the probability distribution of the number of students that are in favor of the notebook requirement?

b) (3 points)

What is the expected number of students, out of the 9 interviewed by the Dean, who will be in favor of the notebook requirement?

c) (3 points)

What is the standard deviation of the number of students, out of the 9 interviewed by the Dean, who will be in favor of the notebook requirement?

d) (3 points)

What is the standard deviation of the percentage of students out of the 9 interviewed by the Dean, who will be in favor of the notebook requirement?
e) (5 points)

What is the probability that 7 or more of the 9 students will be in favor of the notebook requirement?

f) (8 points)

Suppose the Dean becomes more ambitious and samples a total of 40 students.

What is the probability that $7/9 = 77.78\%$ or more of the 40 students interviewed by the Dean will be in favor of the notebook requirement?

*Hint*: Use the normal approximation to the binomial.
Question 3) Election and Gambling (25 points)

This question involves two separate scenarios:

a) (15 points)

In the presidential election in November 2004, the vote was very close. In fact George Bush received 51% of the vote compared to John Kerry’s 49%.

Suppose you conduct a random poll of 10 randomly selected voters and ask them if they voted for George Bush.

i) (10 points)

What is the probability that between 4 and 6 (inclusive) of them voted for George Bush?

ii) (5 points)

Approximately, how different will the percentage who voted for George Bush (from the poll) be from the 51% in the population that voted for George Bush?
b) (10 points)

The probability of winning a certain game of chance is 0.25. Suppose that you decide to play the game 625 times. Assume that successive plays of the game are independent.

What is the probability that you will have 125 or fewer winners?
Question 4) Government Employee Drug Use (25 points)

Illegal drug usage by state government employees can disrupt their lives and lead to poor performance on the job. In Seattle, the usage of (illegal) drugs has been reliably found to be 10% of the state employees. They are very easy to detect in a blood sample, and the test is infallible.

You wish to aid the state government in the screening of a large number of employees for illegal drug use. As an efficiency measure, they combine blood samples from three employees. The combined blood sample is then tested for illegal drugs. If this sample turns out to be negative, then three people are cleared at once. The blood sample will show illegal drug use if at least one of the people is using illegal drugs.

a) (7 points)

Consider any randomly chosen group of three employees. Give the individual probabilities that 0, 1, 2 or 3 of them is using illegal drugs.

b) (3 points)

What is the probability that a combined blood sample from three people will show evidence of illegal drug use?

c) (3 points)

If a combined blood sample turns up positive all three people are tested. What is the expected number of people tested from each combined blood sample?
d) (4 points)

The State is thinking of combining blood samples from four, rather than three people. What is the probability that a combined blood sample from four people will show evidence of illegal drug use?

e) (4 points)

What is the expected number of people re-tested from a combined blood sample?

f) (4 points)

Would you suggest your company increase its combining to four people from three? Justify your answer with reference to the above analysis.
Question 5) New Parks and Reinvigorating neighborhoods (25 points)

This question involves two separate calculation and estimation scenarios:

a) (12 points)

The King County government is thinking of buying a group of buildings in Seattle and converting the space they cover to a park. They are concerned that the value of the buildings is less than the asking price. To aid in the decision-making they are able to obtain independent valuations on 15 of the buildings in the group. The rule that the government has chosen to decide whether they will purchase the entire group of buildings is the following: “Randomly select 15 buildings. If the sum of the 15 individual valuations exceeds 4.8 million dollars, then buy the group. Otherwise do not buy the group.”

If the true distribution of valuations of the buildings in the group is approximately normal with mean $\mu = 0.36$ million dollars and standard deviation $\sigma = 0.08$ million dollars, what is the probability that the government will buy the group, and hence create the park?
b) (13 points)

You represent a foundation that funds small community groups who are trying to reinvigorate city neighborhoods. Only about one in sixteen (0.0625) of the funded projects produce results that reinvigorate a neighborhood. Suppose you independently fund 80 groups.

Using the normal approximation to the binomial, what is the probability that this will result in more than eight neighborhoods being reinvigorated?
Question 6) IRS random audits  (25 points)

The Internal Revenue Service wants to investigate the amount of tax payed by former UW social science students via its offices in Seattle and Issaquah. The IRS audits 10 randomly selected students from the thousands of students reporting to the Seattle office. The first row of the data below are the amounts of tax payed (in thousands of dollars). The second row reports the amounts payed by 9 randomly selected students from the Issaquah office.

Seattle (X)  23 13 12 31 23 24 3 46 39 26
Issaquah (Y)  7 10 4 6 11 21 12 13 6

The sample means and standard deviations are:

\[
\begin{align*}
\text{Seattle} & \quad \bar{X} = 24.00 \\
\text{Issaquah} & \quad \bar{Y} = 20.00 \\
\sigma_X & = 12.78 \\
\sigma_Y & = 5.15
\end{align*}
\]

Note that these are the standard deviations of the populations of students not the standard errors of the means.

a)  (6 points)

What is the standard error for the sample mean tax payed to the Seattle office.

b)  (3 points)

Which of the three distributions: normal, uniform or binomial do you think will provide the best approximation to the shape of the taxes payed? Justify your answer?

c)  (9 points)

Using your choice of distribution, compute the proportion of students from the Seattle office that will pay less than $20,000 in tax?
d) (7 points)

The office wants to construct an upper bound on the tax payed by students to the Seattle office. Find the level of tax payed that only 1% of the students will exceed.
Some numbers

If $X$ is binomial with $n = 100$ and $p = 0.25$ then the Prob. that $X$ is exactly 50 is 0.0000.

If $X$ is binomial with $n = 100$ and $p = 0.5$ then the Prob. that $X$ is exactly 50 is 0.0796.

If $X$ is binomial with $n = 625$ and $p = 0.25$ then the Prob. that $X$ is exactly 125 is 0.0005.

If $X$ is binomial with $n = 625$ and $p = 0.25$ then the Prob. that $X$ is at most 124 is 0.0014.

If $X$ is binomial with $n = 625$ and $p = 0.25$ then the Prob. that $X$ is at most 125 is 0.0019.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the left of 12.78 is 0.19.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the right of 20 is 0.6229.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the left of 20 is 0.3771.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the left of 10 is 0.8633.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the left of 10 is 0.1367.

If $X$ is normal with mean = 0.36 and Std. Dev. = 0.08 then the Area to the left of 0 is 0.0.

If $X$ is normal with mean = 0.36 and Std. Dev. = 0.08 then the Area to the left of 0.3 is 0.2266.

If $X$ is normal with mean = 0.36 and Std. Dev. = 0.08 then the Area to the left of 0.4 is 0.6915.

If $X$ is normal with mean = 0.36 and Std. Dev. = 0.02 then the Area to the left of 0.32 is 0.0228.

If $X$ is normal with mean = 0.36 and Std. Dev. = 0.02 then the Area to the left of 0.42 is 0.9987.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the left of 53.73 is 0.99.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the right of 24 is 0.5.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the left of 2.979 is 0.05.

If $X$ is normal with mean = 24 and Std. Dev. = 12.78 then the Area to the right of
45.92 is 0.05.