Name: Ronald Aylmer Fisher

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<td>Total</td>
<td></td>
<td>100</td>
<td></td>
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</tbody>
</table>
Question 1) Wrestling (25 points)

Television wrestlers Big Belly Stevens and Biceps Bronowski have agreed to compete in a contest at throwing a bowling ball. Each person will get one throw, and the one who throws the longer distance will win.

It has been determined that Stevens’ throws follow a normal distribution with mean 25 meters and standard deviation 2.5 meters. Bronowski’s throws also follow a normal distribution, with mean distance 24 meters and standard deviation 3.0 meters.

Let $X$ be the (random) throw of Stevens and $Y$ the (random) throw of Bronowski.

a) (6 points)

What is that probability that Stevens’ throw will exceed 27 meters?

**Solution**: Now

$$P(X > 27) = P\left(\frac{X - 25}{2.5} > \frac{27 - 25}{2.5}\right)$$

$$= P\left(Z > \frac{27 - 25}{2.5}\right)$$

$$= P\left(Z > 0.80\right)$$

$$= 1 - P\left(Z \leq 0.80\right)$$

$$= 1 - 0.7881$$

$$= 0.2119$$

That is only about 21% of the time.

b) (6 points)

What is the probability that Bronowski’s throw will exceed 27 meters?

**Solution**: Now

$$P(Y > 27) = P\left(\frac{Y - 24}{3.0} > \frac{27 - 24}{3.0}\right)$$

$$= P\left(Z > \frac{27 - 24}{3.0}\right)$$

$$= P\left(Z > 1.00\right)$$

$$= 1 - P\left(Z \leq 1.00\right)$$

$$= 1 - 0.8413$$

$$= 0.1587$$

That is only about 16% of the time.
c) (7 points)

If \( X \) and \( Y \) are independent normal random variables, then \( X - Y \) is also a normal random variable. Moreover, the expected value of \( X - Y \) is 
\[
\mathbb{E}(X) - \mathbb{E}(Y) = 25 - 24 = 1 \text{ meter},
\]
the difference of the expected values. Also, the standard deviation of \( X - Y \) is
\[
\sigma_{X-Y} = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{2.5^2 + 3.0^2} = 3.91 \text{ meters}
\]

If Stevens and Bronowski each make one throw, what is the probability that Stevens will win?

**Hint:** Note that the winner is determined by whether \( X - Y > 0 \) or \( X - Y < 0 \).

**Solution:** Just follow the hint. Stevens wins when \( X - Y > 0 \). But \( X - Y \) has mean 1 and standard deviation 3.91. Then
\[
P(X - Y > 0) = P\left(\frac{X - Y - 1}{3.91} > \frac{0 - 1}{3.91}\right)
= P\left(Z > -0.256\right)
= P\left(Z \leq 0.256\right)
= 0.6009
\]
That is, the probability that Stevens will win is 0.6009.

d) (6 points)

Does there exist a value \( c \) such that 
\[
P(\text{Steven’s throw is more than } c) = P(\text{Bronowski’s throw is more than } c)?
\]

If so, find \( c \).

**Hint:** This is harder.

**Solution:** Just decree the equality \( P(X > c) = P(Y > c) \) and then standardize. You get
\[
P(X > c) = P(Y > c)
= P\left(\frac{X - 25}{2.5} > \frac{c - 24}{2.5}\right) = P\left(\frac{Y - 24}{3.0} > \frac{c - 24}{3.0}\right)
= P\left(Z > \frac{c - 25}{2.5}\right) = P\left(Z > \frac{c - 24}{3.0}\right)
\]
These will be equal when
\[
\frac{c - 25}{2.5} = \frac{c - 24}{3.0}
\]
This equation can be re-expressed as
\[
6(c - 25) = 5(c - 24)
\]
or
\[
c = 30.
\]
The probability is 0.0228 for both men, but you were not asked for this.
Question 2) Bond Barometer (25 points)

As a guide for investment the Wall Street Journal publishes information on the total returns for various categories of bonds. The data below compares a prediction of the total return for the next twelve months (based on the last quarter) to return over the previous twelve months for 17 categories of bonds.

Assume that these bonds are a random sample from the population of all bonds available.

<table>
<thead>
<tr>
<th>Fund Category</th>
<th>Previous Year</th>
<th>Prediction for the next Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Coupon Bonds</td>
<td>24.9</td>
<td>28.8</td>
</tr>
<tr>
<td>Convertible Bonds</td>
<td>21.3</td>
<td>20.4</td>
</tr>
<tr>
<td>Long-Term U.S. Treas.</td>
<td>15.1</td>
<td>18.1</td>
</tr>
<tr>
<td>Global Bonds</td>
<td>11.6</td>
<td>17.3</td>
</tr>
<tr>
<td>General Muni. Bonds</td>
<td>13.1</td>
<td>13.4</td>
</tr>
<tr>
<td>Flexible Income Funds</td>
<td>11.8</td>
<td>12.6</td>
</tr>
<tr>
<td>Longer-Term U.S. Govt.</td>
<td>10.0</td>
<td>10.8</td>
</tr>
<tr>
<td>Intermediate Corporate</td>
<td>9.5</td>
<td>10.8</td>
</tr>
<tr>
<td>Intermediate U.S. Treas.</td>
<td>9.3</td>
<td>10.2</td>
</tr>
<tr>
<td>Intermediate U.S. Govt.</td>
<td>8.0</td>
<td>8.9</td>
</tr>
<tr>
<td>High-Yield Junk</td>
<td>15.4</td>
<td>7.8</td>
</tr>
<tr>
<td>Short-Term U.S. Treas.</td>
<td>5.6</td>
<td>6.6</td>
</tr>
<tr>
<td>Short-Term Corporate</td>
<td>5.9</td>
<td>6.4</td>
</tr>
<tr>
<td>Short-Term U.S. Govt.</td>
<td>5.6</td>
<td>6.2</td>
</tr>
<tr>
<td>GNMA</td>
<td>6.7</td>
<td>4.3</td>
</tr>
<tr>
<td>Adjust. Rate Mort.</td>
<td>4.1</td>
<td>3.9</td>
</tr>
<tr>
<td>Short-Term Global Inc.</td>
<td>3.8</td>
<td>2.2</td>
</tr>
<tr>
<td>mean</td>
<td>10.69</td>
<td>11.10</td>
</tr>
<tr>
<td>standard deviation</td>
<td>5.90</td>
<td>6.87</td>
</tr>
</tbody>
</table>

a) (6 points)

Construct a 95% confidence interval for the mean total return for the previous year.

Solution: A 95% confidence interval for the total return, \( \mu_{1992} \), of the population of bonds for the previous year is

\[
(\bar{X}_{1992} - t_{\alpha/2}(n-1) \times \frac{s_{1992}}{\sqrt{n}}, \ \bar{X}_{1992} + t_{\alpha/2}(n-1) \times \frac{s_{1992}}{\sqrt{n}})
\]

\[
= (\bar{X}_{1992} - t_{0.025}(16) \times \frac{s_{1992}}{\sqrt{n}}, \ \bar{X}_{1992} + t_{0.025}(16) \times \frac{s_{1992}}{\sqrt{n}})
\]

\[
= (10.69 - 2.120 \times \frac{5.90}{\sqrt{17}}, \ 10.69 + 2.120 \times \frac{5.90}{\sqrt{17}})
\]

\[
= (7.66\%, \ 13.72\%)
\]

as \( \bar{X}_{1992} = -0.77, \ n = 17, \ s_{1992} = 5.90, \ \alpha = 0.05. \)
b) (7 points)

Construct a 99% confidence interval for the mean total return for the prediction of the next year.

Solution: A 99% confidence interval for the predicted total return, \( \mu_{1993} \), of the population of bonds for the next year is

\[
\left( \bar{X}_{1993} - t_{\alpha/2}(n-1) \times \frac{s_{1993}}{\sqrt{n}}, \, \bar{X}_{1993} + t_{\alpha/2}(n-1) \times \frac{s_{1993}}{\sqrt{n}} \right)
\]

\[
= (\bar{X}_{1993} - t_{0.005}(16) \times \frac{s_{1993}}{\sqrt{n}}, \, \bar{X}_{1993} + t_{0.005}(16) \times \frac{s_{1993}}{\sqrt{n}})
\]

\[
= (11.10 - 2.921 \times \frac{6.87}{\sqrt{17}}, \, 11.10 + 2.921 \times \frac{6.87}{\sqrt{17}})
\]

\[
= (6.23\%, \, 15.97\%)
\]

as \( \bar{X}_{1993} = 11.10, \, n = 17, \, s_{1993} = 6.87, \, \alpha = 0.01. \)

c) (5 points)

Based on the results of a) and b), do you think the mean total return for the previous year and the mean prediction of the total return for the next year are identical? Briefly say why.

Solution: The claim is that predicted total return, \( \mu_{1993} \), of the population of bonds for the next year is equal to the total return, \( \mu_{1992} \), of the population of bonds for the previous year. If they are equal then their confidence intervals should have substantial overlap. We see that the confidence intervals are almost identical (noting that one is 95% and the other is 99%). As such, there is little evidence in the data that the mean total return for the previous year and the mean prediction of the total return for the next year are different. That is, we do not reject the hypothesis.

d) (7 points)

One Wall Street pundit claims that the mean total return for the bonds for the next year will be over 12%.

Based on your results in part b), do you think this pundit’s claim is consistent with the data on the predictions for next year?

Solution: Expressing, as above, the predicted total return of the population of bonds for the next year by \( \mu_{1993} \), the claim is that this value is over 12%. As the above 99% confidence interval for \( \mu_{1993} \) includes 12%, there is little evidence in the data that the mean total return for the bonds for the next year is not 12% as claimed. Note that even though 12% falls in the upper range of the interval estimate, this is not evidence, in and of itself, that the population mean is not 12%.