

Case-based Social Statistics I

CSSS 322

Professor: Mark S. Handcock

Solutions to Homework 1

Due Tuesday, April 9, 2002

Problems to be handed in:

- 1) Exercise 10.12 from page 308 in Chapter 10: “Estimating Proportions with Confidence” of MOS.

a)

Solution(1): As we will be more confidence of covering the unknown mean, the interval will be wider. ∞

b)

Solution(1): As we will be less confidence of covering the unknown mean, the interval will be decreased. ∞

- 2) Exercise 10.23 from page 309 in Chapter 10: “Estimating Proportions with Confidence” of MOS.

Solution(3): The sample proportion is $\hat{p} = 57/300 = 0.19$. The 95% confidence interval is $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. The calculation is then $0.19 \pm \sqrt{\frac{0.19(1-0.19)}{300}}$ which is 0.145 to 0.235. ∞

- 3) Exercise 10.25 from page 309 in Chapter 10: “Estimating Proportions with Confidence” of MOS.

a)

Solution(2): The sample proportion is $\hat{p} = 0.59$. The standard error is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. The calculation is then $\sqrt{\frac{0.59(1-0.59)}{1000}} = 0.01555$ or 1.555%. ∞

b)

Solution(2): The 95% confidence interval is $\hat{p} \pm 1.96s.e.\hat{p}$. The calculation is then $0.59 \pm 1.96 * 0.01555$ which is 0.56 to 0.62, or 56% to 62%. ∞

- 4) Submit electronically exercises 3, 4, and 5 from Unit B11 of *CyberStats* entitled “Sampling Distributions.”

- 5) Submit electronically exercises 2, 3, 5, 6, 7, 10, and 13 (all are related) from Unit B12 of *CyberStats* entitled “Central Limit Theorem.”

Extra Credit Problem:

- 6) A Washington State Circuit Judge with two courts and serious transportation constraints wants to keep the total workday at the down-town Seattle court down to six hours. The cases that are presented at this court require an amount of time which is approximately normally distributed with mean 32 minutes and standard deviation 12 minutes. Find the maximum number of cases, n , that can be scheduled while keeping the total time under six hours, with probability at least 95%.

hint: This problem has several solution methods. Trial-and-error is probably the easiest way. Talk with other students about how this might be solved!

Solution(5): Let X be the average time, in minutes, needed by n cases. Our required condition is simply $P(X < \frac{360}{n}) \geq 0.95$.

The distribution of X is approximately normal with mean $\mu = 32$ minutes and with standard deviation $\frac{12}{\sqrt{n}}$. It follows that

$$P(X < \frac{360}{n}) = P\left(\frac{X - 32}{\frac{12}{\sqrt{n}}} < \frac{\frac{360}{n} - 32}{\frac{12}{\sqrt{n}}}\right) = P\left(Z < \frac{\frac{360}{n} - 32}{\frac{12}{\sqrt{n}}}\right)$$

and we want this to be at least 0.95. Since $P(Z < 1.645) = 0.95$ (using the normal table, Table A.1, **MOS**, p. 539), we have to solve for n in

$$\frac{\frac{360}{n} - 32}{\frac{12}{\sqrt{n}}} = 1.645$$

This is a quadratic equation in \sqrt{n} the positive solution of which is $\sqrt{n} = 3.060$. That is,

$$n = 9.36$$

Actually, we want the largest value of n which produces a value below 1.645. So we should take $n = 9$.

Alternatively, it is easy enough to proceed by trial and error.

If $n = 10$, then $(360/10 - 32)/(12/\sqrt{10}) = 1.054$. Since $P(Z < 1.054)$ is about 85%, it appears that $n = 10$ is too big.

If $n = 9$, then $(360/9 - 32)/(12/\sqrt{9}) = 2.00$. Since $P(Z < 2.00)$ is about 98%, it appears that $n = 9$ is about right.

If $n = 8$, then $(360/8 - 32)/(12/\sqrt{8}) = 3.06$. Since $P(Z < 3.06)$ is about 99.9%, it appears that $n = 8$ is much too small.

That is, the judge should schedule a maximum of 9 cases at her down-town court room to ensure her workday at most six hours, with 95% confidence.

∞