

Case-based Social Statistics II

CSSS 322

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Solutions to Homework 3

Due Thursday, April 25, 2002

Problems to be handed in:

- 1) In a random sample of 725 selected for interview from a population of 13,916 teachers in Washington, 113 said they thought that the amount of standardized testing was retarding the learning of students in their classes.

- a) Find the best estimate of the percentage of all teachers in Washington who are dissatisfied.

Solution(3): The sample percentage of teachers who are dissatisfied is $f = \frac{113}{725} = 15.59\%$. Because this is a random sample, this is the best estimate the percentage of all teachers in Washington who are dissatisfied. ∞

- b) Find the standard error of your estimate of the percentage of all teachers who are dissatisfied.

Solution(3): The standard error of the estimate is $S_f = \sqrt{f(1-f)}/\sqrt{n} = 1.347\%$. ∞

- c) Find the best estimate of the overall number of teachers who are dissatisfied.

Solution(2): The best estimate is $13,916 \times 0.1559 = 2,169$ dissatisfied teachers. That is the estimated proportion by the total. ∞

- d) Find a 95% confidence interval for the percentage of dissatisfied teachers.

Solution(3): The confidence interval formula is:

$$f \pm t_{\alpha/2}(n-1) \times s_f$$

Here $\alpha = 0.05$ so the interval is:

$$\begin{aligned} &0.1559 \pm 1.96 \times 0.01347 \\ &=(12.95\%, 18.23\%) \end{aligned}$$

Note that as $n > 40$ we approximate the t -multiplier by that with infinite degrees of freedom, 1.96. ∞

- e) In implementing the policy of standardized testing, the state wants to keep the percentage of dissatisfied teachers at or below 10%. Could this reasonably be the case, or do you have convincing evidence that the percentage is larger than 10%? Justify your answer with reference to a test.

Solution(3): No. This percentage (15.59%) is significantly higher than 10%. The reference value (10%) is not in the confidence interval from part d. The t statistic is $(0.559 - 0.1)/0.01347 = 4.15$. That is 0.10 is approximately 4.15 standard deviations below the best estimate. ⊗

2) The bakery at the QFC in University Village produces loaves of bread with “1 pound” written on the label. Here are weights of randomly sampled loaves from the production last week:

1.02, 0.97, 0.98, 1.10, 1.00, 1.02, 0.98, 1.03, 1.03, 1.05, 1.02, 1.06

The mean is $\bar{X} = 1.0216$ and the sample standard deviation is $s = 0.0371$.

a) Find a 95% confidence interval for the mean weight of all loaves produced last week.

Solution(3): The confidence interval is then

$$\bar{X} \pm t_{\alpha/2}(n - 1) \times \frac{s}{\sqrt{n}}$$

Here $\alpha = 0.05$, $\bar{X} = 1.0216$, $s = 0.0371$ so the interval is:

$$1.0216 \pm 2.201 \times \frac{0.0371}{\sqrt{12}}$$

$$1.0216 \pm 0.0236$$

that is (0.998, 1.045) ⊗

b) State the null and alternative hypotheses.

Solution(3): The hypotheses are $H_0 : \mu_0 = 1$ and $H_1 : \mu_0 \neq 1$. ⊗

c) Perform the hypothesis test at the $\alpha = 0.05$ significance level.

Solution(3): Do not reject H_0 as a reasonable possibility. The observed average weight (1.022 pounds) is not significantly different from 1 pound because the confidence interval (from part a) includes 1. The t statistic is 2.02. ⊗

d) What error, if any, might you have committed?

Solution(3): As we do not reject, we may be committing an error of not rejecting a null hypothesis that is in fact false. This is a Type II error. ⊗

3) The owner of a Downtown parking lot suspects that the person she hired to run the lot is stealing some money from the receipts. The receipts as provided by the employee indicate that the average number of cars parked in the lot is 125 per day and that, on average, each car is parked 3.5 hours. In order to determine whether the employee is stealing, the owner watches the lot for 5 days. On those days the number of cars parked is as follows:

120, 130, 124, 127, 128

For the 629 cars that the owner observed during the 5 days, the mean and the standard deviation of the time spent on the lot were 3.6 and 0.4 hours, respectively.

- a) What are the two ways that the employee can be stealing? For each of these ways, can the owner conclude at the 5% level of significance that the employee is stealing?

Solution(6): Let μ_{cars} be the expected number of cars per day parked at the lot. Let μ_{hours} be the expected number of hours per day for the typical car parked at the lot. In the former case, the alternative hypothesis ("guilty verdict") is that the expected number of cars per day parked at the lot is greater than 125. The null or established hypothesis ("innocent verdict") is that the expected number of cars per day parked at the lot is equal to 125. Again think about the court room analogy here. In symbols:

$$H_0 : \mu_{cars} = 125$$

$$H_1 : \mu_{cars} > 125$$

From the sample $\bar{X}_{cars} = 125.8$ and $s_{cars} = 3.90$. We have $n = 5$ days of data. The 5% level of significance (i.e. 95% level of confidence) has $\alpha = 0.05$. The confidence interval is then

$$\begin{aligned} \bar{X}_{cars} \pm t_{\alpha}(n-1) \times \frac{s_{cars}}{\sqrt{n}} \\ 125.8 \pm 2.132 \times \frac{3.90}{\sqrt{5}} \\ 125.8 \pm 3.72 \end{aligned}$$

that is (122.1, 129.5)

Note that the confidence interval, corresponds to a one-sided test rather than a two-sided test. We do not reject the null hypothesis as 125 falls inside the confidence interval. On this basis there is insufficient evidence, at the 5% significance level, to support the owner's beliefs about the number of cars.

In the latter case, the alternative hypothesis ("guilty verdict") is that the expected number of hours per day for the typical car parked at the lot is greater than 3.5. The null or established hypothesis ("innocent verdict") is that the expected number of hours per day for the typical car parked at the lot is equal to 3.5. In symbols:

$$H_0 : \mu_{hours} = 3.5$$

$$H_1 : \mu_{hours} > 3.5$$

From the sample $\bar{X}_{hours} = 3.6$ and $s_{hours} = 0.4$. We have $n = 629$ cars of data. The 5% level of significance (i.e. 95% level of confidence) has $\alpha = 0.05$. The confidence interval is then

$$\begin{aligned} \bar{X}_{hours} \pm t_{\alpha}(n-1) \times \frac{s_{hours}}{\sqrt{n}} \\ 3.6 \pm 1.645 \times \frac{0.4}{\sqrt{629}} \\ 3.6 \pm 0.026 \end{aligned}$$

that is (3.57, 3.63)

Note that the confidence interval, corresponds to a one-sided test rather than a two-sided test. We reject the null hypothesis as the null value (3.5) falls outside the confidence interval and the confidence interval falls in the region specified by the alternative hypothesis. On this basis there is sufficient evidence, at the 5% significance level, to support the owner's beliefs about the number of cars.

✕

b) What are the meanings of Type I and Type II errors in this case?

Solution(1): A Type I error is to decide that the employee is guilty of stealing when in fact the employee is innocent. A Type II error is to decide that the employee is innocent of stealing when in fact the employee is guilty. In the former case the employee may file a wrongful dismissal suit. In the latter case the employee will not be punished for the crime, and may indeed continue to steal. \times

c) If you are the owner, do you want a small or large confidence level (i.e small level of significance, α). Why?

Solution(1): Based on the interpretation given in b) the owner should have α large to have the largest chance of detecting if the employee is stealing. However the owner should be concerned about not making α too large because it will increase the possibility that the employee may file a wrongful dismissal suit. \times

d) If you are the employee, do you want a small or large confidence level (i.e small level of significance, α). Why?

Solution(1): Based on the interpretation given in b) the employee should have α small to have the largest chance of escaping detection. This is especially true if the employee is guilty. However the employee should be concerned about not making α too small because it will increase the possibility that the owner will conduct another test because he feels that the level of evidence required was too high.

As an aside it there is strong evidence that the employee is cheating on the number of hours, but not on the number of cars. \times

4) Your work for social support network that is considering a new system of connecting clients to consolors and wishes to test if the new contact times are significantly different, on average, than the existing your current system. From past records it is established that the mean contact time of the current system is 2.38 days. A test of the new system shows that, with 48 clients, the average contact time was 1.91 days with a sample standard deviation of 0.43 days.

a) Identify the null and alternative hypotheses for a two-sided test, using both words and mathematical symbols.

Solution(3): The null hypothesis, $H_0: \mu = 2.38$, claims that the new system's mean time is the same as for the current system. The research hypothesis, $H_1: \mu \neq 2.38$, claims that is different. \times

b) Perform a two-sided test at the 5% significance level and describe the result.

Solution(3): The average is $\bar{X} = 1.91$ days with standard deviation $s = 0.43$ days and $n = 48$. The confidence interval is then

$$\bar{X} \pm t_{\alpha/2}(n - 1) \times \frac{s}{\sqrt{n}}$$

Here $\alpha = 0.05$ so the interval is:

$$1.91 \pm 1.96 \times \frac{0.43}{\sqrt{48}}$$

$$1.91 \pm 0.843$$

that is (1.79, 2.03)

We reject the null hypothesis as the null value (2.38) falls outside the confidence interval. On this basis there is sufficient evidence, at the 5% significance level, to support the belief that the new system is different than the old system, and based on the mean is probably better. \boxtimes

- c) Perform a two-sided test at the 1% significance level and describe the result.

Solution(3): The confidence interval is then

$$\bar{X} \pm t_{\alpha/2}(n-1) \times \frac{s}{\sqrt{n}}$$

Here $\alpha = 0.01$ so the interval is:

$$1.91 \pm 2.576 \times \frac{0.43}{\sqrt{48}}$$

$$1.91 \pm 0.160$$

that is (1.75, 2.07)

We reject the null hypothesis as the null value (2.38) falls outside the confidence interval. On this basis there is sufficient evidence, at the 1% significance level, to support the belief that the new system is different than the old system, and based on the mean is probably better. \boxtimes

- d) State the p -value as either $p > 0.05$, $p < 0.05$, $p < 0.01$, or $p < 0.001$.

Solution(1): Based on b) and c), the p -value is less than 0.001. \boxtimes

- e) Summarize the results in a brief paragraph to your fellow workers.

Solution(3): To a large extent this is an exercise. The key phrase should be something like “ ... The new contact system is significantly faster than our current system ($p < 0.001$), reducing the time from our current average of 2.38 days down to an estimated 1.91 days ...” \boxtimes