Review of Mathematics for Social Scientists
CSSS 505
Professor: Mark S. Handcock

Solutions to Homework 5

Due Thursday, May 30, 2002

1) The average level of air pollution in Seattle varies during the typical summer day as follows:

$$p(t) = \begin{cases} 2 + 4t & \text{if } 0 \leq t < 2 \\ 6 + 2t & \text{if } 2 \leq t < 4 \\ 14 & \text{if } 4 \leq t < 12 \\ 50 - 3t & \text{if } 12 \leq t < 16 \end{cases}$$

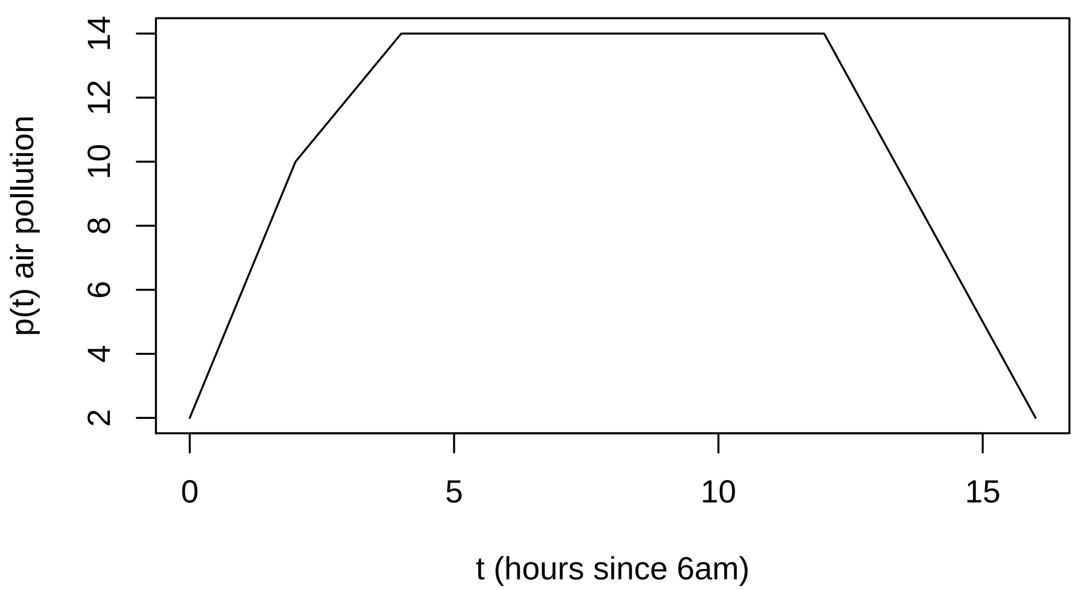
Here t is the time in hours after 6am (and so $t = 16$ corresponds to 10pm).

a) Is $p(t)$ a function (of t)? What are the range and domains of $p(t)$?

Solution(1) : Yes, $p(t)$ a function (of t), with range $[2, 14]$ and domain $[0, 16)$ $\bowtie$

b) Graph the function either by hand or on a computer.

Solution(1) : Here is the plot:



$\bowtie$

c) What are the pollution levels at 8am, 12 noon, 6pm and 8pm?

Solution(1) : This is for $t = 2, 6, 12,$ and $t = 14,$ respectively. From the graph or direct calculation these are $10, 14, 14, 8,$ respectively. ⋈

2) The population of a suburb in Arizona at time t years after 1995 is given by the function:

$$p(t) = 10000 + 1000t - 120t^2$$

a) What is the change in population from $t = 3$ and $t = 5$?

Solution(1) : The population at time $t = 3$ is $p(3) = 11920$, and the population at time $t = 5$ is $p(5) = 12000$. Hence the change in population is $p(5) - p(3) = 80$. ⋈

b) What is the average rate of growth in population from $t = 3$ and $t = 5$? That is the average change in population per year.

Solution(1) : Since the total change is 80, the average change per year is $80/2 = 40$ per year. ⋈

c) What is the average rate of growth in population from $t = 3$ and $t = 3.25$?

Solution(1) : Similarly, this is $(p(3.25) - p(3))/(3.25 - 3) = (11982.5 - 11920)/0.25 = 250$. ⋈

d) What is the average rate of growth in population from t and $t = t + \Delta t,$ where Δt is a fraction of a year?

Solution(1) : This is,

$$\begin{aligned}\frac{p(t+\Delta) - p(t)}{t + \Delta - t} &= \frac{(10000 + 1000(t+\Delta) - 120(t+\Delta)^2) - (10000 + 1000t - 120t^2)}{\Delta} \\ &= \frac{1000\Delta - 120t^2 - 240t\Delta - 120\Delta^2 + 120t^2)}{\Delta} \\ &= 1000 - 240t - 120\Delta\end{aligned}$$

Note that for small values of Δ this is $1000 - 240t,$ the derivative of $p(t)$. ⋈

3) What are the derivatives of the following functions with respect to the independent variables involved.

a) $f(x) = 3x^2 + 1$

Solution(2) : $\frac{df(x)}{dx} = 6x$ ⋈

b) $f(u) = \frac{1}{1+u}$

Solution(2) : $\frac{df(u)}{du} = \frac{-1}{(1+u)^2}$ ⋈

c) $f(u) = \frac{u}{1+u}$

Solution(2) : $\frac{df(u)}{du} = \frac{1}{(1+u)^2}$ ⋈

d) $f(p) = 1/\sqrt{p}$

Solution(2) : $\frac{df(p)}{dp} = \frac{1}{2p^{3/2}}$ ⋈

e) The population function $p(t)$ in Question 2.

Solution(2) : $\frac{dp(t)}{dt} = 1000 - 240t$. ⋈

f) $f(x) = (3x^2 + 1)^7$

Solution(2) : $\frac{df(x)}{dx} = 7(3x^2 + 1)^6 \times 6x = 42x(3x^2 + 1)^6$. ⋈

g) $f(x) = xe^x$

Solution(2) : $\frac{df(x)}{dx} = e^x + xe^x = (1 + x)e^x$. ⋈

h) $f(x) = e^7$

Solution(2) : $\frac{df(x)}{dx} = 0$, as $f(x)$ does not depend on x . ⋈

l) $f(u) = u^2 \log(u)$

Solution(2) : $\frac{df(u)}{dx} = 2u \log(u) + u^2 \times \frac{1}{u} = 2u \log(u) + u$. ⋈

j) $f(u) = x^2 \log(x)$

Solution(2) : $\frac{df(u)}{du} = 0$, as $f(u)$ does not depend on x . ⋈

4) What are the (indefinite) integrals of the following functions with respect to the independent variables involved.

a) $f(x) = 3x^2 + 1$

Solution(2) : $\int f(x)dx = x^3 + x + C$ ⋈

b) $f(u) = u \log(3)$

Solution(2) : $\int f(u)dx = 0.5u^2 \log(3) + C$ ⋈

c) The population function $p(t)$ in Question 2.

Solution(2) : $\int p(t)dt = 10000t + 500t^2 - 40t^3 + C$ ⋈

d) $f(x) = e^7$

Solution(2) : $\int f(x)dx = xe^7 + C$ $\bowtie$

e) $f(u) = \log(u)$

Solution(2) : $\int f(u)dx = u\log(u) - u + C$ as the derivative of $u\log(u) - u + C$ is just $\log(u)$. $\bowtie$