1. (20 points) Let \( A = \{2, 3, 4, 6, 9, 10\} \) and \( B = \{2, 3, 7, 10\} \) be two sets, contained within the sample space \( S \) consisting of the integers from 1 to 100 inclusive. Describe the set \( C = (A^c \cap B) \cup (A \cap B^c) \) in terms of its elements. How would this set \( C \) be affected if we let \( S \) consist of the interval \([0, 100]\)?

\[
B \cap A^c = \{7\}, \quad A \cap B^c = \{4, 6, 9\} \quad \implies \quad C = (A^c \cap B) \cup (A \cap B^c) = \{4, 6, 7, 9\}
\]

2. (20 points) We have two boxes. Box 1 contains 2 red balls and a blue ball. Box 2 also contains 3 balls, with respective colors blue, red and white. You select a box at random and from that box you select a ball at random. Find

\[
P(\text{box 2 was chosen} | \text{a blue ball was selected}) = \frac{P(\text{Box 2} | B)}{P(B)} = \frac{P(\text{Box 2} \cap B)}{P(B)} = \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{1}{2}
\]

\[
P(\text{box 2 was chosen} | \text{a white ball was selected}) = \frac{P(\text{Box 2} \cap W)}{P(W)} = \frac{\frac{1}{3} \cdot \frac{1}{2} + 0 \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}} = 1
\]

\[
P(\text{box 2 was chosen} | \text{a red ball was selected}) = \frac{P(\text{Box 2} \cap R)}{P(R)} = \frac{\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{1}{3}
\]
3. **(20 points)** The discrete random variable $X$ takes on the possible values $-1$ and $+1$ with respective probabilities $0.6$ and $0.4$. Find $EX$ and $\text{var} \ X$.

\[
\begin{array}{c|c|c|c|c}
    x & p(x) & xp(x) & x^2 & x^2p(x) \\
    \hline
    -1 & 0.6 & -0.6 & 1 & 0.6 \\
    1 & 0.4 & 0.4 & 1 & 0.4 \\
    \hline
    & -0.2 & 1.0 & & \\
\end{array}
\]

$\implies EX = -0.2$ and $\text{var} \ X = E(X^2) - (EX)^2 = 1.0 - (-0.2)^2 = 0.96$.

4. **(10 points)** Four female and two male rowers are randomly split into two groups of 4 and 2 to fill a quad and a double, respectively, without regard to seat order. What is the chance that the double (a 2 person boat) will consist of a man and a woman? Give the answer as a reduced fraction (e.g., $3/5$ and not $6/10$).

There are \( \binom{6}{2} = 6 \cdot 5/(1 \cdot 2) = 15 \) ways to choose two persons for the double, and of those choices exactly \( \binom{4}{1} \cdot \binom{2}{1} = 8 \) consist of 1 woman and 1 man. Hence the desired chance is $8/15$.

5. **(10 points)** In how many different orders can I present 5 distinct exam problems to my class?

\[5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120 \text{ ways}\]

6. **(10 points)** What will the response in R be when given the command `choose(5,2)`?

\[\text{choose}(5, 2) = \binom{5}{2} = \frac{5 \cdot 4}{1 \cdot 2} = 10\]

7. **(10 points)** Describe in words what \texttt{R} computes when given the command `1-pbinom(10,100,.5)`.

It computes the chance of observing at least 11 heads in 100 flips of a fair coin

\[1 - \text{pbinom}(10,100,.5) = 1 - P(X \leq 10) = P(X \geq 11)\]

where $X$ is the number of successes in $n = 100$ trials with success probability $p = 1/2$. 