In the card game cribbage you get a hand of 6, dealt from an ordinary deck of 52 cards. From those 6 cards you must discard 2 and keep 4. The same applies for your opponent. The object is to get as many points, when counting from your hand, as possible. You also get points in other ways while playing your hand, but that is not under consideration here. For the purpose of this discussion you get 2 points for any pair of matching value cards, say for 2 sevens, but also 2 points for any group of cards that sum up to 15, where face cards count as 10.

There is also an up-card that can be used as a 5th card by either player when counting the value of a player’s hand together with that 5th card.

Furthermore, when you have a jack in your hands and a card with same suit as that jack turns up, you can get a point for that “right” jack. You also get points for other card patterns, but that is immaterial in this discussion.

It is well known that 29 is the highest possible count in a cribbage hand (with the up-card). It is realized when you get 3 fives and a jack (with different suit from those 3 fives) in your hand (you would discard the other 2) and the 4th five is the up-card (same suit as the jack, the right jack!).

The count to 29 is as follows: There are \( \binom{4}{3} = 6 \) pairs of fives for 12 points. There are \( \binom{4}{3} = 4 \) ways to have three fives add to 15, for 8 points, 4 ways to have a jack/five combination add to 15, for 8 points, and finally 1 point for having the right jack on your hand, for a total of 12 + 8 + 8 + 1 = 29 points.

What is the chance of getting such a hand and up-card combination?

The order of dealing out the cards (although well prescribed in the rules of cribbage) is of no consequence. Thus we can focus on the hand of six dealt to the lucky player and the up-card. There are

\[
\binom{52}{6} \cdot \binom{46}{1} = 936491920
\]

such deals

Of these deals there are

\[
\binom{4}{3} \cdot 1 \cdot \binom{47}{2} \cdot 1
\]

deals giving a count of 29
Here $\binom{4}{3}$ counts the ways to get 3 fives, 1 counts the jack with the complementary suit, and $\binom{47}{2}$ counts the number of ways to draw an additional 2 cards (to get a hand of 6) avoiding the five that’s still out there, and the last 1 accounts for the remaining five as up-card. Thus

$$P(29 \text{ hand}) = \frac{\binom{4}{3} \cdot \binom{47}{2}}{\binom{52}{6} \cdot \binom{46}{1}} = \frac{(4 \cdot 47 \cdot 46/2) \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46} = \frac{1}{26 \cdot 17 \cdot 10 \cdot 49} = \frac{1}{216580}$$

See the discussion below about treating this as odds of 1 in 216580 against a 29 hand.

The next highest hand in cribbage is 28 which you get when you hold all four fives in your hand and any ten value shows as up-card, or you have 3 fives in your hand with the fourth five as up-card, but you must have a ten value card in your hand. Here the counting is a bit more complicated and depends on what you do when you get the “right jack” (different suit from the 3 fives, when you just have 3 fives) and another ten value in your hand.

A) If you have and keep the right jack, then you should not count the deals that lead to a 29 count.

B) If you have only the right jack and no other ten value in your hand, then you cannot get a 28 hand (but you could get 29). If you have the right jack and another ten value, then opt for keeping one of the other ten values and discard the right jack. That avoids the possibility of a 29 hand.

First we will count all deals giving 28 when you disregard the possible single point that could result from a right jack in the hand. After having obtained that count we subtract the 29 hand deals under strategy A) and B).

4 fives and no ten values in hand, a ten value turns up:

$$\binom{4}{4} \cdot \binom{32}{2} \cdot \binom{16}{1} = 7936 \text{ ways}$$

4 fives and exactly one ten value in hand, another ten value turns up:

$$\binom{4}{4} \cdot \binom{16}{1} \cdot \binom{32}{1} \cdot \binom{15}{1} = 7680 \text{ ways}$$

4 fives and two ten values in hand, another ten value turns up:

$$\binom{4}{4} \cdot \binom{16}{2} \cdot \binom{14}{1} = 1680 \text{ ways}$$

3 fives and exactly one ten value in hand and 4th five in up-card

$$\binom{4}{3} \cdot \binom{16}{1} \cdot \binom{32}{2} = 31744 \text{ ways}$$
3 fives and exactly two ten values in hand and 4th five in up-card

\[
\left( \begin{array}{c} 4 \\ 3 \end{array} \right) \cdot \left( \begin{array}{c} 16 \\ 2 \end{array} \right) \cdot \left( \begin{array}{c} 32 \\ 1 \end{array} \right) = 15360 \quad \text{ways}
\]

3 fives and three ten values in hand and 4th five in up-card

\[
\left( \begin{array}{c} 4 \\ 3 \end{array} \right) \cdot \left( \begin{array}{c} 16 \\ 3 \end{array} \right) = 2240 \quad \text{ways}
\]

for a total of 66640 deals. Of those some will give a count of 29 depending on A) or B).

Under A) the only deals that lead to 29 are those where you get 3 fives, the right jack, and any other two values (besides the 4th five) in your hand, and the 4th five comes as up-card.

There are

\[
\left( \begin{array}{c} 4 \\ 3 \end{array} \right) \cdot \left( \begin{array}{c} 47 \\ 2 \end{array} \right) = 4324 \quad \text{ways}
\]

thus 66640 \( - \) 4324 = 62316 deals resulting in a count of 28. Thus the probability of getting a 28 hand deal is \( p = 62316/936491920 = .00006654195 \) and the odds against a 28 hand are \( (1-p)/p = (936491920-62316)/62316 = 15027.11 \), while the value of odds against is usually given\(^1\) as 15028, which appears to be a rounded version of 936491920/62316 = 15028.11, which in turn does not exactly conform to the definition of odds\(^2\).

Under B) the only deals that lead to 29 are those where you get 3 fives, the right jack and any other two non-ten values in your hand, and the 4th five comes as up-card. There are

\[
\left( \begin{array}{c} 4 \\ 3 \end{array} \right) \cdot \left( \begin{array}{c} 32 \\ 2 \end{array} \right) = 1984 \quad \text{ways}
\]

thus 66640 \( - \) 1984 = 64656 deals resulting in a count of 28. Thus the probability of getting a 28 hand deal is \( p = 64656/936491920 = .00006904064 \) and the odds against a 28 hand are \( (1-p)/p = (936491920-64656)/64656 = 14483.22 \), while the value of odds against is usually given as 14484, which appears to be a rounded version of 936491920/64656 = 14484.22, which again does not exactly conform to the definition of odds.

Based on this, the odds against a 29 hand should really be cited as 216579 to 1. We showed the decimals in the other odds as contrast to this case where we deal with true integers.

\(^1\)http://www.cribbageforum.com/Mailbag5.htm
\(^2\)http://en.wikipedia.org/wiki/Odds