2. Ross, problem 29, page 89.
5. Ross, problem 76, page 95.

6. Let $A, B, C$ be independent random variables uniformly distributed on $(0, 1)$. What is the probability that $Ax^2 + Bx + C$ has real roots?

7. (a) Suppose that $X$ is distributed according to a Poisson distribution with parameter $\lambda$. The parameter $\lambda$ is itself a random variable whose distribution law is exponential with mean $1/c$: $f_{\lambda}(t) = c \exp(-ct)1_{[0,\infty)}(t)$. Find the distribution of $X$.
   (b) What if $\lambda$ follows a Gamma distribution of order $\alpha$ with scale parameter $c$: i.e. $f_{\lambda}(t) = c(ct)^{\alpha-1} \exp(-ct)/\Gamma(\alpha)$ for $t > 0$?

8. Let $X_1, X_2$ be independent random variables with uniform distribution over the interval $[\theta - 1/2, \theta + 1/2]$. Show that $X_1 - X_2$ has a distribution independent of $\theta$ and find its density function.

9. Using the central limit theorem for suitable Poisson random variables, prove that

$$
\lim_{n \to \infty} e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!} = \frac{1}{2}.
$$

10. Suppose we have $N$ chips numbered $1, 2, \ldots, N$. We take a random sample of size $n$ without replacement. Let $X$ be the largest number in the random sample. Show that the probability mass function of $X$ is

$$
P(X = k) = \left(\frac{k-1}{n-1}\right) \left(\frac{N}{n}\right), \text{ for } n, n + 1, \ldots, N,
$$
and that

\[ E(X) = \frac{n}{n+1} (N + 1), \quad Var(X) = \frac{n(N - n)(N + 1)}{(n + 1)^2(n + 2)}. \]