Reminder: Midterm exam 1, Monday, October 28
Math/Stat 394 Probability Review notes at

Reading: Ross; Chapter 4, pages 191 - 230
        Durrett; Chapter 5, 185-207

Due: Wednesday, October 30, 2013.

1. Durrett, chapter 1, page 75: problem 1.2.
   Five white balls and five black balls are distributed in two urns in such a way
   that each urn contains five balls. At each step we draw one ball from each urn
   and exchange them. Let $X_n$ be the number of white balls in the left urn at time
   $n$. Compute the transition matrix for $X_n$.

2. Durrett, chapter 1, page 75: problem 1.5.
   Consider a gambler’s ruin chain (as in Durrett, Example 1.1, pages 1,2, Example
   1.11, page 11) with $N = 4$. That is, if $1 \leq i \leq 3$, $P(i, i+1) = 0.4$ and $P(i, i-1) =
   0.6$, but the endpoints are absorbing states $P(0, 0) = 1$ and $P(4, 4) = 1$. Compute
   $P^3(1, 4)$ and $P^3(1, 0)$.

   A taxicab driver moves between the airport $A$ and two hotels $B$ and $C$ according
   to the following rules. If he is at the airport, he will be at one of the two hotels
   next with equal probability. If at a hotel, then he returns to the airport with
   probability $3/4$ and goes to the other hotel with probability $1/4$.
   (a) Find the transition matrix for the chain.
   (b) Suppose the driver begins at the airport at time 0. Find the probability for
   each of his three possible locations at time 2 and the probability he is at hotel $B$
   and time 3.

4. Durrett, chapter 1, page 84: problem 1.45:
   Consider a general chain with state space $S = \{1, 2\}$ and write the transition
   probability matrix as
   $$
P = \begin{pmatrix}
   1 - a & a \\
   b & 1 - b
   \end{pmatrix}
$$
where $0 < a < 1$ and $0 < b < 1$. Use the Markov property to show that

$$P(X_{n+1} = 1) - \frac{b}{a + b} = (1 - a - b) \left\{ P(X_n = 1) - \frac{b}{a + b} \right\},$$

and then conclude

$$P(X_n = 1) = \frac{b}{a + b} + (1 - a - b)^n \left\{ P(X_0 = 1) - \frac{b}{a + b} \right\}.$$

This shows that if $0 < a + b < 2$, then $P(X_n = 1)$ converges exponentially fast to its limiting value, $a + b$. 