Homework 1.
Due Wednesday, January 14.

1. Let $X_1, \ldots, X_n$ be iid $U(q-\frac{1}{2}, q+\frac{1}{2})$. Let $T = (U, V) = (\min X_i, \max X_i)$ and $S = V - U$.
   (a) Derive the conditional distribution of $T$ given that $S = s$.
   (b) Show that a 95% conditional confidence interval for $q$ given $S = s$ is $\frac{1}{2}(v + u) \pm 0.475(\frac{1}{s})$. When is this the same as the unconditional one derived in class?

2. Let $X_1, \ldots, X_n$ be iid $N(q, c^2)$ where $c > 0$ and $c$ is a known positive constant.
   Show that $A = \sum X_i \left( \frac{1}{n} \sum X_i^2 \right)^{1/2}$ is ancillary.

3. Show that for $X_1, \ldots, X_n$ iid $N(\mu, \sigma^2)$
   (a) any empirical moment around the sample mean is distributed independently of the sample mean;
   (b) the quadratic form $X_1 A X$ is distributed independently of $X$ if and only if $A$ has all row sums zero;
   (c) the sample range is distributed independently of the sample mean;
   (d) $(X_{(n)} - \bar{X})/\sigma_{X_{(n)}}$ is distributed independently of the sample mean and the sample variance.

4. Suppose measurements are taken by either of two instruments, with observations $(A_i, X_i), i = 1, \ldots, n$, where $A_i = 1$ (instrument 2 used) $\sim$ Bern($\frac{1}{2}$), and $X_i | A_i \sim N(q, \sigma^2_{A_i})$.
   (a) Find the mle of $\mu$.
   (b) What is the Fisher information about $\mu$ in the sample?
   (c) Show that $A = \sum A_i$ is ancillary.
   (d) Show that the conditional variance of $\hat{q}$ given $A = a$ is the reciprocal of the observed information, or $(\sum A_i^2)^{-1}$.