HOMEWORK SET 2
Due January 21

1. Consider a situation with sufficient statistic $(T, A)$, where $T$ is the mle of $\theta$ and $A$ is ancillary for $\theta$.
   (a) Show that the Fisher information about $\theta$ from observing $T$ only is no larger than the Fisher information about $\theta$ from observing the entire sample.
   (b) Show that, if $I_{T\mid A}(\theta)$ is the Fisher information about $\theta$ in the conditional distribution of $T$ given $A$, then in the long run no information about $\theta$ is lost by conditioning on $A$ in the sense that $\int_{T\mid A} I(\theta) f_A(a) da = I(\theta)$.

2. Let $X \sim U(\theta - 1, \theta)$.
   (a) Show that $\lfloor X \rfloor$, the integer part of $X$, can be thought of as an estimate of $\theta$.
   (b) Show that $\Theta(\theta) = X - \lfloor X \rfloor$ is ancillary.
   (c) Find the conditional distribution of $\lfloor X \rfloor$ given $\Theta(\theta)$.
      Hint: When is $\lfloor X \rfloor = \lfloor \theta \rfloor$?

3. In order to assess the precision of a new measuring device, pairs of measurements are taken on a variety of objects. The measurement errors are assumed iid $N(0, \sigma^2)$, so the measured values are $X_{ij} \sim N(\theta, \sigma^2), i = 1, ..., n, j = 1, 2$.
   (a) Show that the mle of $\sigma^2$ converges in probability to $\sigma^2/2$.
   (b) Let $\hat{\sigma}$ be the mle of $\sigma$. Are the $\hat{\sigma}$ ancillary?
   (c) Find the conditional distribution of $X_{ij}$ given $\hat{\sigma}$ and show that the conditional mle of $\sigma^2$ is consistent.