Homework 7.

Due Monday March 8

1. Let $T(F) = \mu_F^2 = (E_F X)^2$. Using the asymptotic theory for statistical functionals, find the limiting distribution of $T(F_n)$ when $\mu_F = 0$.

2. Let $X_1, \ldots, X_n$ be iid nonnegative random variables with cdf $F(x) = \int_0^x \mu dG(t)$
   where $G$ is a cdf on $[0, \infty)$ with mean $\mu$.
   (a) Show that $E\left(\frac{1}{X}\right) = \frac{1}{\mu}$.
   (b) Show that $G(x) = \int_0^x \frac{\mu}{y} dF(y)$.
   (c) Use the results of (a) and (b) to develop an estimator of $G$ and determine its asymptotic distribution.

3. Let $X_1, \ldots, X_n$ be iid from a distribution with density
   
   \[ f(x; \theta) = \left(\frac{x}{\theta}\right)^{\alpha(\theta)} e^{A(\theta) + B(x)}, \ x > 0, \ \theta > 0, \ \text{where} \ A \ \text{is twice differentiable}. \]
   (a) Show that the mle of $\theta$ is the geometric mean.
   (b) Find the influence curve of the mle in (a), sketch it, and show that it is unbounded.
   (c) Can you apply the asymptotic theory for statistical functionals to this estimator?