Solutions to Homework 5
Due Tuesday, February 15, 2005

Problems to be handed in:

1) The probability that a person living in Seattle uses public transport regularly is 0.4. Suppose you are told that a particular person lives close to a bus stop. Would you expect the conditional probability that this person uses public transport regularly given that they live close to a bus stop to be less than, equal to, or greater than the (unconditional) probability of using public transport (i.e. 0.4)?

Solution: It seems logical that a person with more access to public transportation will use it more than a person with less access. So the extra information that a person lives near a bus stop, when added to existing knowledge, will increase the probability of using public transportation. So the probability a person uses public transportation given he lives near a bus stop will be greater then 0.4.

2) The phone operators at the UW Student Counseling Center receive many different types of calls, but especially those that ask for information, and to schedule visits to the Center. The Center would prefer students visit the Center rather than just ask for information over the phone. Requests for information account for 75% of all calls, while 15% of calls schedule a visit to the Center. Also, 10% of calls involve both information requests and scheduling a visit to the Center.

a) What is the conditional probability that a call leads to scheduling a visit, given that it requested information? This tells the Center about the value of providing information by phone to get students to visit the Center.

Solution: It helps to define some notation. Let \( A = \{ \text{information is requested} \} \) and \( B = \{ \text{a visit is scheduled} \} \). The \( A \cap B \) is the event that information is requested and a visit is scheduled. From the description we have:

\[
P(A) = 0.75 \quad P(B) = 0.15 \quad P(A \cap B) = 0.1
\]

Note that the description could be interpreted to be \( A \cap \overline{B}, B \cap \overline{A}, A \cap B \) with similar different numbers below. The conditional probability that a call leads to a visit scheduled given that in formation is requested is

\[P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.75} = 0.1333\]

b) What is the conditional probability that a call did not request information, given that the student scheduled a visit to the Center? This represents the fraction of visits that were natural or unassisted.
Solution (3): The conditional probability that information is requested given a call schedules a visit is

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.15} = 0.6666 \]

Thus the conditional probability that information is not requested given a visit is scheduled is

\[ P(\overline{A}|B) = 1 - \frac{P(A \cap B)}{P(B)} = 1 - 0.6666 = 0.3333 \]

\[ \checkmark \]

c) What is the probability that a call scheduled a visit and do not request information? Interpret this number.

Solution (3): The probability that a call scheduled a visit is

\[ P(B) = P(A \cap B) + P(\overline{A} \cap B) \]

so

\[ P(\overline{A} \cap B) = P(B) - P(A \cap B) \]

This can be done with Venn diagrams also. Thus

\[ P(\overline{A} \cap B) = 0.15 - 0.1 = 0.05 \]

This is, in a sense, a ‘good’ call as it schedules a visit without the overhead of an information request. Note that only 5% of calls are of this kind.

\[ \checkmark \]

d) Why are the answers to to parts b and c different?

Solution (2): They are different because part b (a conditional probability) is the proportion of those that scheduled visits, while part c (an unconditional probability) is the proportion of all calls.

\[ \checkmark \]

e) Are the two events “requested information” and “scheduled a visit” independent? How do you know?

Solution (3): They are not independent as the information that a call scheduled a visit changes the probability that the call requested information. More precisely, from parts a and b respectively:

\[ P(A|B) = 0.6666 \quad P(A) = 0.75 \]

\[ \checkmark \]

3) Exercise 8.11 from page 286 in Chapter 8: “Random Variables” of MOS.

a) Write the simple events in the sample space. Use B for boy and G for girl.

Solution (2): The woman will continue to have children until either she has a girl or she has four children (or both). The sample space is thus \( S = \{G, BG, BBG, BBBG, BBBBB\} \).
b) Find the probability for each of the simple events in the sample space.

Solution(2): Since \( P(B) = P(G) = 0.5 \) and the probability of having a child of a given gender is independent of the gender of the other children, we can compute the probabilities in the sample space as such:

\[
P(G) = 0.5
\]

\[
P(BG) = P(B) \times P(G) = 0.5^2 = 0.25
\]

\[
P(BBG) = P(B) \times P(B) \times P(G) = 0.5^3 = 0.125
\]

\[
P(BBBG) = P(B) \times P(B) \times P(B) \times P(G) = 0.5^4 = 0.0625
\]

\[
P(BBBBB) = P(B) \times P(B) \times P(B) \times P(B) = 0.5^4 = 0.0625
\]

This assigns a probability to each element of the sample space, so we’re done. As a quick check, you can verify that the probabilities add up to 1.

\( \Box \)

c) Find the probability distribution function for \( X \).

Solution(2): Recall that \( X = \) the number of children the woman has. She will have either 1, 2, 3, or 4 children, so those are the possible values for \( X \). The events BBBG and BBBB both fall into the \( X = 4 \) category.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>0.5</td>
<td>0.25</td>
<td>0.125</td>
<td>0.125</td>
</tr>
</tbody>
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d) Draw a picture of the probability distribution function for \( X \).

Solution(2): This is attached at the end.

\( \Box \)
$P(X = x)$
4) Submit electronically exercises 16, 17, 18 and 19 from Unit B6 of CyberStats entitled “Expected Values and Variance.”

a) B6 Ex. 16. What is the probability that a 50-year-old woman does not die during the year she is 50?

Solution: There is a \( 29/10,000 = 1/10,000 = 0.003 \) chance that a 50-year-old woman will die during the year that she is 50, so there is a 1-0.0003 = 0.997 chance that such a woman will not die during that year.

b) B6 Ex. 17. Let \( x \) = amount that the insurance company must pay out on a $100,000 life insurance policy for a 50-year-old woman. List the possibilities of \( x \) and give a probability for each value.

Solution: \( x = $100,000 \) if the woman dies naturally, \( x = $200,000 \) if the woman dies accidentally, and \( x = $0 \) if the woman doesn’t die. Thus, the probability distribution looks like this:

\[
\begin{align*}
P(x = $100,000) &= P(\text{woman dies naturally}) = 29/10,000 = 0.0029 \\
P(x = $200,000) &= P(\text{woman dies accidentally}) = 1/10,000 = 0.0001 \\
P(x = $0) &= P(\text{woman doesn’t die}) = 0.997.
\end{align*}
\]

c) B6 Ex. 18. Calculate the expected value of \( x \)

Solution: \( E(x) = $100,000(0.0029) + $200,000(0.0001) + $0(0.997) = $310. \)

d) B6 Ex. 19. Write a sentence interpreting this expected value.

Solution: the mean payout of the life insurance policy is $310.