You are the investment manager for a small NW liberal arts college. You have an endowment of $1,000,000 and wish to find a long-term investment to secure the future of the school. You know of an internet startup venture that succeeds half the time. For each invested $1,

\[ \begin{cases} 
\text{you make $1.60 when the venture succeeds (plus your initial$1 is returned)} \\
\text{you make nothing when the venture fails (plus you lose your initial$1)} 
\end{cases} \]

You may invest in a series of such internet ventures for as long as you like; you may also risk as much as you please.

While this appears to be an attractive one-time investment you run the risk of losing all your money if you are both careless and unlucky at the same time. To avoid the former you develop a Capital Retention Strategy: Always invest exactly half the money in your possession.

For this laboratory we wish to understand the effect of combining such an attractive investment with this capital-retention strategy.

1) What are the outcomes from the first investment? How much do you expect to gain or lose on this first venture?

**Solution** (3): You first invest half of the $1,000,000, $500,000. If the venture fails, you lose it and are left with the remaining $500,000. Your next investment will be $250,000. If the venture succeeds, you make $1.60 \times $500,000 = $800,000 to add to your original $1,000,000 for a total of $1,800,000. Your next investment will be $900,000. As each of these outcomes are equally likely your expected final capital is 

\[ \frac{1}{2} \times$1,800,000 + \frac{1}{2} \times$500,000 = $1,150,000. \] 

Thus you expect to gain 15% on the first venture.

2) If you sequentially invest in 10 ventures like this how much money, on average, would you have at the end? (ball-park figures will do)

**Solution** (3): Since you only bet half your capital each time, you can never go broke, so you can be sure you will survive long enough to make 10 investments. If you start with capital \( C_n \) before the \( n^{th} \) investment, then you will have \( 1.8C_n \) if it succeeds and \( 0.5C_n \) if it fails. Thus

\[ C_{n+1} = \begin{cases} 1.8C_n \text{ with probability } \frac{1}{2} \\
0.5C_n \text{ with probability } \frac{1}{2} \end{cases} \]
As all these ventures are independent:

\[ C_{n+1} = 1,000,000 \times 1.8^S \times 0.5^{n-S} \]

where \( S \) is the number of the \( n \) ventures that are successful. Clearly \( S \) is a binomial random variable with probability of success \( \frac{1}{2} \) over \( n \) ventures. The expected capital after \( n \) ventures is then

\[
\sum_{s=0}^{n} C_{n+1} \times P(S = s) = \sum_{s=0}^{n} 1,000,000 \times 1.8^s \times 0.5^{n-s} \times \left( \begin{array}{c} n \\ s \end{array} \right) \left( \frac{1}{2} \right)^n
\]

\[
= 1,000,000 \left( \frac{1}{2} \right)^2 \sum_{s=0}^{n} \left( \begin{array}{c} n \\ s \end{array} \right) 1^{n-s} 3.6^s
\]

\[
= 1,000,000 \left( \frac{1}{2} \right)^2 (1 + 3.6)^n
\]

\[
= 1,000,000 \times (1.15)^n
\]

Thus over \( n = 10 \) ventures the expected capital at the end is \$4,045,558.

3) If you invest in 10 ventures how many will have to succeed so that you will end up with more than the \$1,000,000 you started with?

**Solution** (3): The probability of being ahead after \( n \) ventures is

\[
P(C_{n+1} > C_1) = P(1.8^S \times 0.5^{n-S} > 1)
\]

\[
= P(S \log(1.8) + (n - S) \log(0.5) > 0) = P\left( \frac{S}{n} > \frac{\log(2)}{\log(3.6)} \right)
\]

Thus the proportion of successes must be over \( \frac{\log(2)}{\log(3.6)} = 0.541126 \). Thus 6 of the 10 ventures will have to succeed so that you will end up with more than the \$1,000,000 you started with.

4) How likely is it that you will end up ahead when your 10th investment is completed?

**Solution** (3): From the previous question, the exact probability is

\[
P(S \geq 6) = \left( \frac{1}{2} \right)^{10} \sum_{s=6}^{10} \left( \begin{array}{c} 10 \\ s \end{array} \right) = 0.377
\]

Thus we will end up ahead only about one third of the time.

5) Would you decide to invest in this venture 10,000 times? Discuss its merits and drawbacks. If you can, suggest a better strategy than the *Capital Retention Strategy*. 
**Solution (3):** Using the formulas developed in the previous questions it is easy to show that over $n = 10,000$ ventures the expected capital at the end is about $10^{613}$. This is an astronomically large figure. However you need to succeed in 5412 of the 10,000 ventures to end up with more than the $1,000,000 you started with. The probability that we will end up ahead decreases exponentially as the number of ventures increases. After $n = 10,000$ ventures it is $0.979087 \times 10^{-17} \approx 10^{-16}$. Thus the probability that we will end up ahead is astronomically small.

In short, the good news is that you can not go broke and you will make an enormous amount of money, on average. The bad news is that you can be almost certain that after 10,000 investment you will have less than what you started with! Part of the reason is that you need to win much more often than you lose to break even.

The issue is that your average return is not your sole consideration. If you repeatedly stake all your capital on one type of investment you may lose everything eventually, no matter how attractive the average return may be. Insurance, for example, is a situation where people buy at unfavorable odds (with a negative average return) to avoid the possibility of a large loss. An insurance company that ensures against fire may decline to insure 100 identical houses in the same region. A bookie who accepts a bet of $100 usually declines one of $100,000 at the same odds. The insurance company and bookie both choose to accept a lower average return to secure their survival.

The moral is that: you can’t judge strategy (the consequences of a sequence of decisions) by just analyzing tactics (each decision in isolation).

A better strategy might be to bet the same amount (e.g. $100,000) on each venture and stopping if the remaining capital is below $150,000. Alternative you could bet it all on one venture. The return, if it succeeds, is $1,600,000. If not, its time to go back to UW - the College is bankrupt!
In this lab, you are going to simulate investing in 10 ventures by tossing a coin 10 times, and keeping track of the successes and failures:

\[
\text{if the coin toss is } \begin{cases} 
\text{heads} & \text{the venture succeeds} \\
\text{tails} & \text{the venture fails}
\end{cases}
\]

The results for the class

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<th>calculated probability</th>
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