18.0 Confidence Intervals and testing for two samples

18.1 Confidence intervals for the difference of two independent samples

Sample from population $X$

$X_1, \ldots, X_m$  mean $m_X$

$6_x$

Sample from population $Y$

$Y_1, \ldots, Y_n$  mean $m_Y$

$6_y$

How do we compare the means?
Investigate the difference between the means

\[ m_x - m_y \]

An estimate, as usual,

What is the expected value of

\[ \bar{x} - \bar{y} \]
We assume $\sigma_x$ and $\sigma_y$ are equal.

If $\sigma_x$ and $\sigma_y$ differ greatly, comparing the two populations by looking at the mean difference is usually poor.

--- $Y$

--- $X$
An estimate of the common variance

\[ s_p^2 = \frac{(m-1)s_x^2 + (n-1)s_y^2}{m+n-2} \]

Remarks

a) \( s_p \) is between \( s_x \) & \( s_y \)

b) \[ s_p^2 = ps_x^2 + (1-p)s_y^2 \]

\( p = \) proportion of total sample size from \( X \)
Using our general rule:
A $100(1-\alpha)\%$ confidence interval for $\mu_x - \mu_y$ is

$$\bar{X} - \bar{Y} \pm t_{\alpha/2} \left( \frac{m+n-2}{m} + \frac{1}{n} \right) s_p \sqrt{\frac{1}{m} + \frac{1}{n}}$$

Remarks

a) $\text{Var}(\bar{X} - \bar{Y}) = \text{Var}(\bar{X}) + \text{Var}(\bar{Y})$

$$= \frac{\sigma_x^2}{m} + \frac{\sigma_y^2}{n}$$

$$= s_p^2 \left( \frac{1}{m} + \frac{1}{n} \right)$$

b) If $m+n-2 \geq 30$ use normal approximation for $t_{\alpha/2} (m+n-2)$, $Z_{\alpha/2}$
Student grades in the fall versus spring quarter

X: Fall  m = 6 students
Y: Spring  n = 8 students

\[ \bar{x} = 74 \quad S_x = 11 \]
\[ \bar{y} = 60 \quad S_y = 9 \]

What is a 95% confidence interval for the difference?
18.2 Confidence intervals for paired samples

Suppose $X$ & $Y$ are the before & after an intervention on the same person.

E.g., $X_i =$ test score for person $i$ in the Fall quarter

$Y_i =$ test score for person $i$ in the Spring quarter

$D_i = X_i - Y_i =$ individual change in test score

Remarks

a) $X_i$ & $Y_i$ are no longer independent

b) can regard $D_i$ as a single sample
A 100(1-\alpha)\% confidence interval for the mean of a paired difference is

\[ \overline{D} \pm t_{\alpha/2}(n-1) \frac{S_D}{\sqrt{n}} \]

Remarks

a) \[ \overline{D} = \overline{X} - \overline{Y} \]

Ex

<table>
<thead>
<tr>
<th></th>
<th>Student</th>
<th>Fall</th>
<th>Spring</th>
<th>( D_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>54</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>54</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>70</td>
<td>89</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>60</td>
<td>77</td>
<td></td>
</tr>
</tbody>
</table>

\[ \overline{D} = \hspace{1cm} S_D = 3.91 \text{ points} \]

A 95\% confidence interval for the mean difference:

\[ t_{0.025} = 3.18 \]
10.3 Comparison of paired samples to independent samples

a) Paired samples work on units with the same characteristics
   Ex: students in Fall & Spring
   - effects of extraneous variables are filtered out

b) Pairing increases the precision of an experimental survey
   That is, the width of the paired confidence interval is less than that of the independent interval

Rule of thumb: Use pairing when possible
10.4 Testing the difference between two means

Suppose we wish to test the hypothesis:

\[
H_0 : \mu_x = \mu_y \\
H_1 : \mu_x \neq \mu_y
\]

As before, calculate the \( t \)-statistic

\[
t = \frac{\bar{x} - \bar{y}}{S_p \sqrt{\frac{1}{m} + \frac{1}{n}}}
\]

Result: At the \( 100(1-\alpha)\% \) level of confidence,

reject \( H_0 \) if \( |t| > t_{\alpha/2} (m+n-2) \)
do not reject \( H_0 \) if \( |t| \leq t_{\alpha/2} (m+n-2) \)
Ex. Quality of Computer Monitors

We test computer 18" monitors for clarity.

Sony

\[
\begin{align*}
84, & 82 \\
84, & 78, 79
\end{align*}
\]

\[ m = \]

\[ S_p^2 = \]

\[ t = \]

\[ t_{0.025}(9) = 2.262 \]

\[ t_{0.025}(10) = 2.228 \]

Radio Shack

\[
\begin{align*}
75, 77, 75 \\
81, 77, 78
\end{align*}
\]

\[ n = \]
18.5 What have we learned

a) Calculating confidence intervals for the difference of two means

b) Confidence intervals for paired samples

c) Comparing paired to unpaired sample schemes

d) For testing the difference between means.