Review of Mathematics for Social Scientists
CSSS 505
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Homework 4
Due Tuesday, May 14, 2002

1) Give the sizes of the following matrices:
   a) 
   \[ A = \begin{bmatrix} 4 & 1 & 3 \end{bmatrix} \]
   b) 
   \[ B = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \end{bmatrix} \]

2) In Exercise 1, if \( B = [b_{ij}] \), find \( b_{11}, b_{22}, b_{21}, b_{23} \) and \( b_{13} \).

3) Give an example of a \( 3 \times 3 \) matrix \([c_{ij}]\) for which \( c_{ij} = -c_{ji}, i, j = 1, 2, 3 \). Note that the diagonal must be zero.

3) International Trade
The trade between three East Asian countries during 2000 (in millions of U.S. dollars) is given by the matrix \( A = [a_{ij}] \), where \( a_{ij} \) represents the exports from the \( i \)th country to the \( j \)th country.

\[ A = \begin{bmatrix} 0 & 16 & 20 \\ 17 & 0 & 18 \\ 21 & 14 & 0 \end{bmatrix} \]

The trade between the same countries during 2001 is given by the matrix \( B \).

\[ B = \begin{bmatrix} 0 & 17 & 19 \\ 18 & 0 & 20 \\ 24 & 16 & 0 \end{bmatrix} \]

a) Write a matrix representing the total trade between the three countries for the two-year period 2000 and 2001.

b) In 2000 and 2001, 1 U.S. dollar was equal to 5 Hong Kong dollars, write the matrix representing the total trade between the three countries for the two-year period in Hong Kong dollars.

4) Perform the indicated operations and simplify:
   a) 
   \[ \begin{bmatrix} 2 & 0 & 1 \\ \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ 3 & 0 \end{bmatrix} \]
b) \[
\begin{bmatrix}
2 & 1 \\
0 & 2 \\
3 & -1
\end{bmatrix}
\left(\begin{bmatrix}
1 & 2 \\
2 & -1
\end{bmatrix} + 3 \begin{bmatrix}
2 & 0 \\
1 & 2
\end{bmatrix}\right)
\]

5) Given

\[
A = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
2 & -1 \\
-3 & -2
\end{bmatrix}
\]

a) Find \((A + B)^2\).

b) Find \(A^2 + 2AB + B^2\).

c) Is \((A + B)^2 = A^2 + 2AB + B^2\)?
6) **Social Networks** A social network consists of a number of points, called actors, some of which are connected by lines that denote a social tie between the actors (e.g., friendship). Two examples of small social networks with four and five actors are given below:

If the actors are numbered 1, 2, ..., we define the matrix $X$ by setting $x_{ij} = 1$ if there is a line connecting the two actors $i$ and $j$ and setting $x_{ij} = 0$ if there is not a line connecting the two actors $i$ and $j$. By convention $x_{ii} = 0$ for each actor $i$.

a) Construct $X$ for each of the above networks.

b) Construct $X^2$ in each case.

c) **Extra credit:** Show that the $(i, j)$th element of $X^2$ gives the number of routes from actor $i$ to actor $j$ that pass through exactly one other actor.

d) **Extra credit:** What do you think that $X^3$ gives?